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Translation

Proceedings of the Ninth All-Union School Seminar on Statistical Hydroacoustics

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PROCEEDINGS OF THE NINTH ALL-UNION SCHOOL SEMINAR ON STATISTICAL HYDROACOUSTICS

Novosibirsk TRUDY DEVYATOY VSESOYUZNOY SHKOLY-SEMINARA PO STATISTICHESKOY GIDROAKUSTIKE in Russian 1978 signed to press 17 Jul 78 pp 1-171

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[Book edited by N. G. Zagoruyko, V. V. Ol'Shevskiy, S. V. Pasechnyy, Mathematics Institute of the Siberian Department of the USSR Academy of Sciences]

[Text] This collection contains the materials of the 9th All-Union School Seminar on Statistical Hydroacoustics organized by the Scientific Council on Hydrophysics under the Presidium of the USSR Academy of Sciences, the Acoustics Institute of the USSR Academy of Sciences, the Oceanology Institute of the USSR Academy of Sciences, the Southern Division of the Oceanology Institute of the USSR Academy of Sciences, the Mathematics Institute of the Siberian Department of the USSR Academy of Sciences, and the Kiev Polytechnical Institute.

The sessions of the seminar were held at the Southern Division of the Oceanology Institute of the USSR Academy of Sciences in Gelendzhik from 29 September to 6 October 1977.

The thematics of the collection include the problems of studying the hydrophysical characteristics of the marine environment and the acoustics phenomena in the ocean; the problems of statistical measurements and simulation; the problems of processing signals against a background of interference, the transmission and processing of information by biological subjects.

The materials of these proceedings are of interest for scientific workers and specialists in the field of hydrophysical research and information processing, postgraduate students, engineers and students in the advanced courses.

UDC 551.466.3

STATISTICAL CHARACTERISTICS OF THE SLOPES OF A WAVE SURFACE

By I. N. Davidan, Ya. M. Kublanov, V. A. Rozhkov, Yu. A. Trapeznikov, pp 3-6

The development of various types of radiophysical instruments based on the reflection of light, sound and radio waves from a wavy sea surface, the use of satellite information about the state of the ocean, the performance of the calculations of the energy exchange parameters between the atmosphere and the water require consideration of differential characteristics of waves -- slopes of the wave surface.

Beginning with the assumption that the wave field $\xi(x,y,t)$ is stationary and uniform, the vector random function $\bar{\eta}(x,y,t)$ characterizing the slopes is also stationary and uniform. In this case the random function $\bar{\eta}(x,y,t)$ scan be determined using the vector of the mathematical expectation, the correlation tensor and the tensor of the spectral density of the slopes.

The probability characteristics of the slopes can be determined by the probability characteristics of the prominences of the wave surface. For the solution of this problem it is necessary to have information about the correlation surface of the waves or know the relation between the spectra of the slopes and the prominences. In the latter case, linearity of the wave model and the satisfiability of the specific type of dispersion relation are proposed. Then the invariant sum of the slope of the spectra is related to the frequency spectrum of the prominences by the function

$$S_{\eta_x}(\omega) + S_{\eta_y}(\omega) = K^2 S_\xi(\omega), \quad (1)$$

and the dispersion relation can be given in the form

$$K = \omega^2/g. \quad (2)$$

The analysis of the experimental data obtained by the results of aerial photographic surveys of the waves, measurements using the Volna buoy and multiwire wave recorders indicates the applicability of the dispersion ratio ([]). In Fig 1 the spectral of the prominences $S_\xi(\omega)$, the slopes $S_{\eta_x}(\omega)$ and the ratio of the wave number k calculated by the dispersion expression (2) to the wave number k' calculated by the function (1) are presented. The closeness of k'/k to 1 indicates the correctness of (2) in a quite broad frequency range.

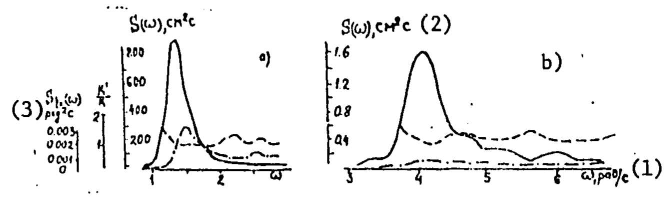

Figure 1. Spectra of the prominences (solid line), spectra of of the slopes (dash-dot line) and k'/k ratio (dashed line); a) -- sea, b) -- lake

Key:

1. ω, rad/sec
2. cm^2sec
3. rad^2sec

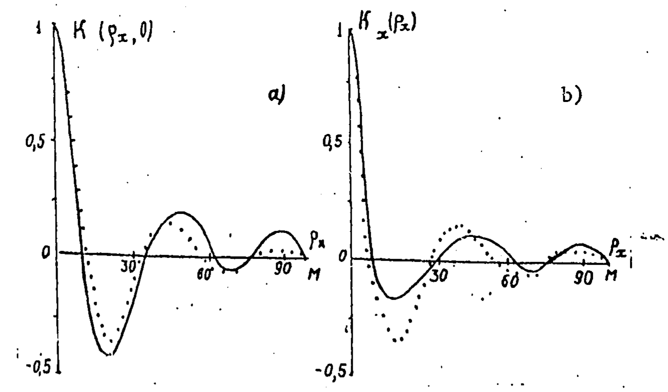

Figure 2. Correlation function of the prominences (a) and the slopes (b) and their approximations (dotted line).

The correlation function of the slopes $K_{\bar{\eta}}(\rho, \beta)$ can be defined in terms of the correlation function $K_\xi(\rho, \beta)$ of the process $\xi(t)$ as the second derivative of $K_\xi(\rho, \beta)$ taken with the inverse sign. As an example, Figure 2 shows the normalized cross section of the correlation surface which was calculated by the aerial photographic survey and its approximation by the expression

$$K_\xi(\rho) = e^{-\alpha|\rho|}\cos\beta\rho. \quad (3)$$

In this figure the corresponding cross section of the correlation surface of the slopes and its approximation calculated by (3) are shown.

The investigated examples indicate that for description of the statistical properties of the slopes it is possible to use the approximated expressions of the spectral density and correlation function of the prominences of the wave surface proposed in reference [2].

The basic experimental methods of determining the slopes of a wavy surface are the aerial photographic survey of the waves, the application of the Volna type buoys or the multiwire wave recorders. When using the survey sheets made during aerial photographic surveys and the sheets from the multiwire wave recorders, the minimum error in determining the slopes of the wave surface can be obtained with an optimal spatial interval of discreteness. The use of the Volna type buoy to determine the slopes presupposes knowledge of it in the transfer functions with respect to the angular role.

The analysis of the experimental data obtained by the results of the aerial photographic survey of the waves, measurements by the Volna buoy and the multiwire wave recorders indicates that with proper selection of the discreteness intervals, reliable data are obtained for calculating the slopes. In Table 1 estimates are presented of the slope dispersion in the general direction of the propagation of wind-driven waves $m_{2\ max}$ calculated by the aerial photographic survey data and wire wave recorders with different spatial discreteness Δx. Inasmuch as for the wind-driven waves the ratio between the average height $\bar{h}$ and the average period $\bar{\tau}$ can be assumed constant. As is obvious from Table 1, when selecting Δx according to the calculations performed in reference [4], the stabilization of the numerical values of $m_{2\ max}$ indicates the reliability of calculating the slopes.

Table 1

Values of $m_{2\ max}$ According to the Data with Different Discreteness of the Aerial Photographic Survey

Δx	24m	12 m	6 m	$\bar{\lambda}$=120 m
$m_{2\ max}$	0.0017	0.0038	0.0056	$\bar{\lambda}/\Delta_x$=5 + 20
Wire wave recorders				
Δx	1.5 m	1 m	0.75 m	$\bar{\tau}$=4 + 5
$m_{2\ max}$	0.0072	0.0069	-	$\bar{\lambda}/\Delta x$ = 20+40
$m_{2\ max}$	0.0068	0.0060	0.0062	

The basic characteristics of the slopes are the events of the two-dimensional spectrum of the waves $m_{2\ max}$, $m_{2\ min}$ which determine the dispersions of the slopes in the general direction of the waves and perpendicular to it, respectively. The dispersion of the slopes in the general direction of propagation of the wave is

$$m_{2max} = 2\pi\left(\frac{\bar{h}}{\bar{\lambda}}\right)^2. \quad (4)$$

The estimates with respect to the natural data of the ratio $m_{2\ max}/m_{2\ min}$, as is illustrated in Fig 3, have great sample variability and are included in the interval from 1.5 to 7.5.

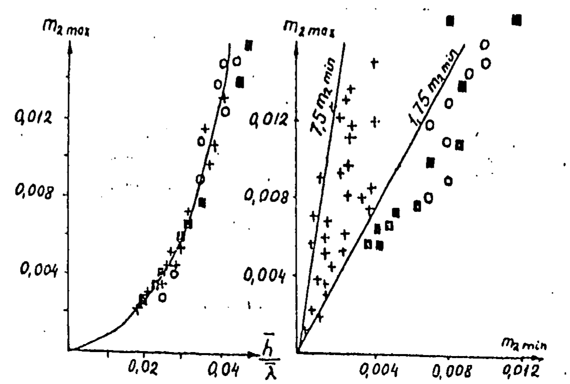

Figure 3. Dispersions of the slopes in the general direction of propagation of the waves $m_{2\ max}$ and $m_{2\ min}$ perpendicular to it according to the aerial photographic survey measurement data (+), the "Volna" buoy (o) and the wire wave recorder (□).

The slope dispersion of the wave surface according to the measurement data is within the limits of 40-80 deg. The instantaneous values of the slopes are found in the range of ±(20-27) deg.

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UDC 551.463.2

PROPERTIES OF ESTIMATES OF THE CHARACTERISTICS OF PERIODICALLY CORRELATED RANDOM PROCESSES AS MODELS OF MARINE WAVE ACTION

By Ya. P. Dragan, I. N. Yavorskiy, pp 7-10

References [1, 2] demonstrate the adequateness of the model of rhythm in the form of a periodically correlated random process (PCRP) with a finite average power to the oscillations of the sea surface at a point. Inasmuch as the ordinary substantiation of finding estimates of the probability characteristics by the results of the statistical processing of the time realizations of the random process is the corresponding version of the hypothesis of ergodicity, it is expedient to find the definition of this property which will encompass the PCRP of the mentioned class as a special case. Let us consider the general class of random processes with finite average power according to R. Forte

$$P_\xi = m_t E|\xi(t)|^2 = m_t r(t,s) < \infty,$$

where $m_t = \lim_{\theta\to\infty} \frac{1}{\theta}\int_0^\theta \cdot\, dt, \quad r(t,s) = E\mathring{\xi}(t)\mathring{\xi}(s), \; \mathring{\xi}(t) = \xi(t) - m_\xi(t),$

$m_\xi(t) = E\xi(t)$. These processes are stationary in the Hilbert space $\mathcal{H}_\mathcal{P}$ with the norm $\|\cdot\|_{\varphi_\mathcal{P}} = (m_t E|\cdot|^2)^{1/2}$.

Definition. Let us call a random process from the class П for which

$$P_{\text{вер}}\{m_t\xi(t) = (\xi(t))_{\varphi_\mathcal{P}}\} = 1, \; P_{\text{вер}}\{m_t\{\mathring{\xi}(t+\tau)\mathring{\xi}(t)\} = \{\mathring{\xi}(t+\tau)\mathring{\xi}(t)\}_{\varphi_\mathcal{P}} = 1$$

П-ergodic.

Theorem 1. For the П-ergodic random process the statistic

$$\hat{B}(\tau) = \lim_{\theta\to\infty} \frac{1}{\theta}\int_0^\theta \mathring{\xi}(t+\tau)\mathring{\xi}(t)\,dt, \quad \hat{m} = \lim_{\theta\to\infty}\frac{1}{\theta}\int_0^\theta \xi(t)\,dt$$

with probability 1 gives an estimate of the mean mathematical expectation and the mean covariation.

Inasmuch as for any function f for which $m|f|<\infty$, we have

$$\lim_{\theta\to\infty}\frac{1}{\theta}\int_0^\theta f(t)dt=\frac{1}{T}\int_0^T\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}f(t+nT)dt,$$

as a corollary of the general theorem 1 we have the theorem:

Theorem 2. For the Π-ergodic process from the class Π^T, that is, the PCRP such that $\frac{1}{T}\int_0^T r(t,t)dt<\infty$, the statistics

$$\hat{m}_\xi(t)=\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\xi(t+nT), \qquad (1)$$

$$\hat{r}(t+\tau,t)=\hat{b}_\xi(t,\tau)=\lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\overset{\circ}{\xi}(t+\tau+nT)\overset{\circ}{\xi}(t+nT), \qquad (2)$$

for almost t give estimates of the mathematical expectation and covariation with a probability of 1.

PCRP. Consequently, the PCRP degenerates into a set of cophasal values on the lattice $\{t+nT,\ n=-\infty,\infty\}$ which are stationary ergodic and stationarily ergodically connected series.

When using the model in the form of a PCRP to study sea waves, an important problem is the investigation of the statistical properties. The estimation of the mathematical expectation (1) for finite N will be unbiased and meaningful when satisfying the condition: $\lim B_\xi(t,\tau)=0$. (3)

The mathematical expectation of the PCRP describes the average wave profile, the shape of which depends on the amplitudes of the harmonics m_ℓ determined by the expansion of this periodic function in the Fourier series:

$$m_\xi(t)=\sum_{\ell=-\infty}^{\infty}m_\ell e^{j\ell\frac{2\pi}{T}t}.$$

The estimates of the amplitudes $\hat{m}_\ell$ can be obtained by the statistics:

$$\hat{m}_\ell=\frac{1}{T}\int_0^T\hat{m}_\xi(t)e^{-j\ell\frac{2\pi}{T}t}dt;\quad \hat{m}_\ell=\frac{1}{\theta}\int_0^\theta\xi(t)e^{-j\ell\frac{2\pi}{T}t}dt.$$

The first of these estimates is unbiased:

$$E\hat{m}_\ell=\frac{1}{T}\int_0^T E\hat{m}_\xi(t)e^{-j\ell\frac{2\pi}{T}t}dt=m_\ell,$$

and the second is asymptotically unbiased:

$$E\hat{m}_\ell=m_\ell+\frac{1}{\theta}\int_{-\alpha T}^{\alpha T}m_\xi(t)e^{-j\ell\frac{2\pi}{T}t}dt-\frac{\alpha}{\theta}\int_0^T m_\xi(t)e^{-j\ell\frac{2\pi}{T}t}dt.$$

Here α is the fractional part of the number θ/T. If the averaging interval is selected a multiple of the correlation period $\Theta=NT$, then it will become unbiased. The dispersions of these estimates are identical and are defined by the expression:

$$\sigma^2_{m_\xi}=\frac{1}{NT^2}\int\int_0^T\sum_{n=-N+1}^{N-1}\left(1-\frac{|n|}{N}\right)B_\xi(t,s-t+nT)e^{j\ell\frac{2\pi}{T}(t-s)}\,dt\,ds.$$

It is obvious that they approach zero for $N\to\infty$ and satisfaction of the condition (3). The estimates of the correlation function by the known mathematical expectation can be determined by the statistics:

$$\hat{B}_\xi(t,\tau)=\frac{1}{N}\sum_{n=0}^{N-1}[\xi(t+nT)-m_\xi(t)][\xi(t+\tau+nT)-m_\xi(t+\tau)], \qquad (4)$$

$$\hat{B}_\xi(t,\tau)=\frac{1}{N}\sum_{n=0}^{N-1}\xi(t+nT)\xi(t+\tau+nT)-m_\xi(t)m_\xi(t+\tau). \qquad (5)$$

Both of these estimates are unbiased, and their dispersions for the gaussian PCRP are defined by the formulas:

$$\sigma^2_{B_\xi}=\frac{1}{N}\sum_{n=-N+1}^{N-1}\left(1-\frac{|n|}{N}\right)[B_\xi(t,\tau+nT)B_\xi(t,\tau-nT)+B_\xi(t,nT)B_\xi(t+\tau,nT)],$$

$$\sigma^2_{B_\xi}=\frac{1}{N}\sum_{n=-N+1}^{N-1}\left(1-\frac{|n|}{N}\right)[B_\xi(t,\tau+nT)B_\xi(t,\tau-nT)+B_\xi(t,nT)B_\xi(t+\tau,nT)+$$
$$+m_\xi(t)m_\xi(t+\tau)(B_\xi(t,\tau+nT)B_\xi(t,\tau-nT))+m^2_\xi(t)B_\xi(t+\tau,nT)+m^2_\xi(t+\tau)B_\xi(t,nT)].$$

It is obvious that the convergence of the second estimate is already somewhat worse, and to a significant degree it is determined by the magnitude of the mathematical expectation.

For the unknown mathematical expectation when it is necessary to estimate it in advance, the statistics (4)-(5) have a nonzero bias error. For the first of them the bias ε_2 is equal to the following:

$$\varepsilon_1=E\hat{B}_\xi(t,\tau)-B_\xi(t,\tau)=-\frac{1}{N}[B_\xi(t,\tau)+\sum_{n=-N+1}^{N-1}\frac{|n|}{N}B_\xi(t,\tau+nT)].$$

It is obvious that it approaches zero for $N\to\infty$, that is, the investigated estimate is asymptotically unbiased. The estimate (5) has the same property. Its bias ε_2 is defined by the formula

$$\varepsilon_2=-\frac{1}{N}\sum_{n=-N+1}^{N-1}\left(1-\frac{|n|}{N}\right)B_\xi(t,\tau+nT).$$

The dispersions of the estimates approaches zero for $N\to\infty$, that is, they are meaningful.

The correlation analysis of the PCRP as models of waves, in addition to obtaining the estimates of the covariations $\hat{B}_\xi(t,\tau)$ permitting the study of its dependence on time and the shear τ also states the problem of finding the estimates of the correlation components $\hat{B}_\ell(\tau)$ with which information about the spectral composition of the process and the correlations between the harmonics are also connected. The estimates of the correlation components are possible by the statistics:

$$\hat{B}_\ell(\tau) = \frac{1}{\theta}\int_0^\theta \mathring{\xi}(t)\mathring{\xi}(t+\tau)e^{-j\ell\frac{2\pi}{T}t}dt, \quad \hat{B}_\ell(\tau) = \frac{1}{T}\int_0^T \hat{b}_\xi(t,\tau)e^{-j\ell\frac{2\pi}{T}t}dt. \qquad (6\text{-}7)$$

both the first and second estimates for θ=NT are unbiased if the mathematical expectation is known.

With unknown mathematical expectation the estimates of the components $B_\ell(\tau)$ and the estimates of the correlation function will be asymptotically unbiased. For finite N the bias of estimate (6) and estimate (7) when the correlation function is found by the formula (4):

$$\sigma'^2_{B_\ell} = \frac{1}{NT^2}\sum_{n=-N+1}^{N-1}\left(1-\frac{|n|}{N}\right)\int_0^T\int_0^T [b_\xi(t,\tau+nT+s-t)b_\xi(t+\tau,s-t-\tau+nT)+$$

$$+b_\xi(t,s-t+nT)b_\xi(t+\tau,s-t+nT)]e^{j\ell\frac{2\pi}{T}(s-t)}ds\,dt$$

coincide and are defined by the formula

$$\varepsilon_3 = E\hat{B}_\ell(\tau) - B_\ell(\tau) = -\frac{1}{N}\left[B_\ell(\tau) - \sum_{n=-N+1}^{N-1}\frac{|n|}{N}B_\ell(\tau+nT)\right].$$

If the correlation function is calculated using expression (5), then the bias of the estimate 10 is equal to:

$$\varepsilon_4 = -\frac{1}{N}\sum_{n=-N+1}^{N-1}\left(1-\frac{|n|}{N}\right)B_\ell(\tau+nT).$$

The analysis of the dispersions of the investigated estimates indicates that they are meaningful. The results obtained in the given paper make it possible to judge the quality of the possible estimates of the mathematical expectation of the covariation of the PCRP as models of the wind-driven waves, indicate that the degree of their approximation to the estimated values depends on the methods of finding their values.

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POSSIBILITY OF ISOLATING A SIGNAL IN THE ENERGY SPECTRUM OF AN ELECTROMAGNETIC WAVE REFLECTED FROM AN OSCILLATING INTERFACE

By V. Yu. Varavin, E. P. Gulin, pp 10-15

The reflection of an electromagnetic wave from an oscillating surface leads to variation of the spectral composition of the scattered field by comparison with the incident one. Here the spectrum of the scattered electromagnetic wave contains information about the nature of the oscillations of the scattering surface. If in addition to the free statistical movements the reflecting water surface completes forced regular oscillations excited by a sound source in the water, the problem arises of the possibility of isolating the component corresponding to the regular oscillations in the spectrum of the field of an electromagnetic wave reflected from the surface. Some estimates as applied to the phase spectrum of the reflected wave for the simplest model of the reflecting surface are contained in references [1, 2].

Let us consider the case of a statistically rough water surface assuming that the curvature of its unevennesses will permit the application of the Kirchhoff approximation to solve the diffraction problem. The prominences of the points of the scattering surface $Z=\xi(\vec{r},t)$ will be represented in the form of the sum of the random component $\xi_{cT}(\vec{r},t)$ subject to normal distribution with zero mean and space-time correlation coefficient [3]

$$\Gamma_\xi(\vec{\rho},\tau)=\frac{2}{\sigma_\xi^2}\int_{-\pi}^{\pi}\tilde{\theta}(\vartheta)\int_0^\infty G(\Omega)\cos[\Omega\tau-\varkappa(\Omega)\rho\cos(\vartheta-\vartheta_0)]\,d\vartheta\,d\Omega, \quad (1)$$

where $\tilde{\theta}(v)$ and $G(\Omega)$ are the angular and frequency spectra of the random prominences respectively, σ_ξ^2 is the dispersion of the random prominences, $\chi(\Omega)$ is the wave number of the surface waves, v_0 is the angle between the separation vector $\vec{\rho}=\vec{r}_1-\vec{r}_2$ and the general direction of propagation of the surface waves at the component $\xi_p(\vec{r},t)$ corresponding to the oscillations of the water surface excited by the sound field in the water:

$$\xi_p(\vec{r},t)=h\sin[\omega_0 t-K\xi_{cT}(\vec{r},t)], \quad (2)$$

where h is the oscillation amplitude, ω_0 is the frequency, K is the wave number of the sound wave.

In the case of matched source and receiver of electromagnetic waves located in the Fraunhofer zone with respect to the scattering surface, the reflected field E_r for normal incidence of the monochromatic wave on the surface can be written in the form

$$E_r(t) = \frac{\kappa V \exp(i2\kappa R_0 - i\omega_1 t)}{4i\pi R_0^2} \int_S \exp[-i2\kappa \xi(\vec{z},t)]\, d^2\vec{z}, \quad (3)$$

where ω_1 and k are the frequency and the wave number of the electromagnetic wave,

V is the reflection coefficient from the water-air interface,

R_0 is the distance from the source (receiver) of the electromagnetic waves to the midplane z=0. Hereafter we shall consider that the linear dimensions of the scattering surface S are large by comparison with the spatial scale of the correlation of the random prominences of the surface.

Inasmuch as the problem consists in distinguishing the regular processing against a background of a random process it is natural to use the spectral representations. Since in practice the duration of the observation, that is, the duration of the realization or the selection of the investigated process is finite, it is necessary to work with sample spectra which in the investigated case are of a stochastic nature. It is known [5] that with a probability of $(1-\alpha)$ 100% , the sample spectrum will lie inside the confidence interval, the boundaries of which are determined by the values of the energy spectrum of the investigated process (here $\alpha<1$ is a positive number given in advance). If the upper bound of the $(1-\alpha_1)$ 100% confidence interval for the sample spectrum of the scattered field in the absence of sound excitation of wave surface does not exceed the lower bound of the $(1-\alpha_2)$ 100% confidence interval for the sample spectrum of the scattered field in the presence of sound excitation, then on introduction of the sample spectral density in the confidence interval for the spectrum in the presence of sound excitation with a probability of $(1-\alpha_2)$ 100% it is possible to state that the scattering surface is excited by the sound. The probability of false alarm with this algorithm does not exceed $\alpha_1 \cdot 50\%$, and the probability of a miss, α_2 50%.

The width of the confidence intervals can be decreased by "smoothing" (averaging over several realizations) of the sample spectra. At the same time the conditions of separation of the confidence intervals for the sample spectra with and without sound excitation of the water surface are improved. It is demonstrated (see, for example, [5]) that the value of $\nu\overline{G}_T(\omega)/\langle G_T(\omega)\rangle$, where $\overline{G}_T(\omega)=\sum_{n=1}^{N} G_{Tn}(\omega)/N$ is a sample spectrum smoothed with respect to N realizations of duration T, and $\langle G_T(\omega)\rangle$ -- the average sample spectrum with respect to the set -- is approximately subordinate to χ^2-distribution with ν degrees of freedom, where ν=3N (for $N\geq 2$). Let us denote by $G_T^c(\omega)$ and $G_T^w(\omega)$ the spectra of the reflected field, correspondingly, in the presence and absence of sound excitation of the surface. Then in the adopted notation the confidence intervals for the smoothed sample spectra will be defined by the expressions:

$$\frac{\langle G_T^c(\omega)\rangle \chi_\nu(\alpha_2/2)}{\nu} \leqslant \bar{G}_T^c(\omega) \leqslant \frac{\langle G_T^c(\omega)\rangle \chi_\nu(1-\alpha_2/2)}{\nu}, \quad (4)$$

$$\frac{\langle G_T^w(\omega)\rangle \chi_\nu(\alpha_1/2)}{\nu} \leqslant \bar{G}_T^w(\omega) \leqslant \frac{\langle G_T^w(\omega)\rangle \chi_\nu(1-\alpha_1/2)}{\nu}, \quad (5)$$

where $\chi_\nu(\alpha_2/2)$ and $\chi_\nu(1-\alpha_2/2)$ are the lower and upper bounds respectively of the $(1-\alpha)$ 100% confidence interval for the random variable subordinate to the normalized χ^2-distribution with ν degrees of freedom. It is obvious that $G_T^w(\omega)=G_T^c(\omega)$ for h=0.

The average sample spectrum $\langle G_T(\omega)\rangle$ the reflected field $E_r(t)$ is related by the known expression to the correlation moment

$$\langle E_z(t)E_z^*(t+\tau)\rangle,$$

$$\langle G_r(\omega)\rangle = \frac{1}{2\pi}\int_0^T\Big[\frac{1}{T}\int_{-T/2}^{T/2-\tau}\langle E_z(t)E_z^*(t+\tau)\rangle dt\Big]\exp(-i\omega\tau)d\tau +$$
$$+\int_{-T}^{0}\Big[\frac{1}{T}\int_{-T/2-\tau}^{T/2}\langle E_z(t)E_z^*(t+\tau)\rangle dt\Big]\exp(-i\omega\tau)d\tau. \quad (6)$$

Using (3), after averaging over the set of random surfaces it is easy to obtain:

$$\langle E_z(t)E_z^*(t+\tau)\rangle = \frac{K^2V^2S}{16\pi^2R_0^4}\exp(i\omega_1\tau)\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}J_n(2kh)\,\times \quad (7)$$
$$\times J_m(2kh)\exp[-in\omega_0 t + im\omega_0(t+\tau)]\int_0^{\Delta x}\int_0^{\Delta y}Q_2(q_n,-q_m,\vec{\rho},\tau)\Big(1-\frac{\rho_x}{\Delta x}\Big)\Big(1-\frac{\rho_y}{\Delta y}\Big)d^2\vec{\rho},$$

where $J_n(2kh)$ is the Bessel function, $Q_2=\langle\exp[-i(q_n\xi_1-q_m\xi_2)]\rangle$ is a two-dimensional characteristic function of the prominences of the surface, $q_n=2k-nk$, $\vec{\rho}=\{\rho_x,\rho_y\}$; as the surface S, a rectangular area is taken with linear dimensions Δx and Δy along the x and y axes.

Substituting (7) in (6) and performing the integration with respect to t under the assumption $\omega_0 T_0 \gg 1$, we obtain

$$\langle G_r^n(\omega)\rangle \simeq \frac{K^2V^2S}{16\pi^3R_0^4}\sum_{n=-\infty}^{\infty}J_n^2(2kh)\int_0^T\int_0^{\Delta x}\int_0^{\Delta y}Q_2(q_n,-q_n,\vec{\rho},\tau)\,\times$$
$$\times\Big(1-\frac{\tau}{T}\Big)\Big(1-\frac{\rho_x}{\Delta x}\Big)\Big(1-\frac{\rho_y}{\Delta y}\Big)\cos[(\omega-\omega_1-n\omega_0)\tau]d\tau\, d^2\vec{\rho}. \quad (8)$$

In the absence of sound excitation, only one term of the sum with $n=0$ differs from zero. Being given the normal distribution of the surface prominences and considering the waviness isotropic (which approximately corresponds to the case of ripples on the surface of the water), for gravity waves ($\chi(\Omega)=\Omega^2/g$) in accordance with (1), under the condition $k^2\sigma_\xi^2>>1$, we have:

$$Q_2 = \exp\{-2K^2\sigma_\xi^2[1-\Gamma_\xi(\vec{\rho},\tau)]\} \simeq \exp[-2K^2\sigma_\xi^2(\overline{\Omega^2}\tau^2+\frac{\overline{\Omega^4}}{2g^2}\rho^2)], \quad (9)$$

where $\overline{\Omega^n}=\frac{2}{\sigma_\xi^2}\int_0^\infty \Omega^n G_\xi(\Omega)d\Omega$, g is the gravitational acceleration. Substituting (9) in (8) and assuming that the spatial scale of the correlation of the scattered field is small by comparison with the linear dimensions of the scattering area and the time scale of the field correlation is small by comparison with the duration of the realization ($K^2\sigma_\xi^2\overline{\Omega^2}T^2>>1$) we obtain:

$$\langle G_r^\omega(\omega)\rangle = \langle G_r^c(\omega)\rangle_{R,0} \simeq \frac{M}{8K^3}\exp[-\frac{(\omega-\omega_1)^2}{8K^2\sigma_\xi^2\overline{\Omega^2}}], \quad (10)$$

where $M=K^2V^2Sg^2/(4\sqrt{2}\pi^{3/2}R_0^4\sqrt{\overline{\Omega^2}}\,\overline{\Omega^4}\sigma_\xi^3)$. In the given case the spectrum of the scattered field has one minimum -- at the frequency of the electromagnetic field $\omega=\omega_1$.

In the presence of sound excitation, the spectral density of the scattered field is a polymodal curve with maxima on the combination frequency $\omega=\omega_1+n\omega_0$, where $n=0\pm1, \pm2, \ldots$. For the n-th term of the series (8) under the condition $q_n^2\sigma_\xi^2\overline{\Omega^2}T^2>>1$, we obtain:

$$\langle G_r^c(\omega)\rangle_n \simeq \frac{M}{q_n^3}J_n^2(2Kh)\exp[-\frac{(\omega-\omega_1-n\omega_0)^2}{2q_n^2\sigma_\xi^2\overline{\Omega^2}}]. \quad (11)$$

The width of the spectral maximum corresponding to the n-th term of the series is equal to $\Delta\omega_n=2\sqrt{2}|q_n|\sigma_\xi\sqrt{\overline{\Omega^2}}$. The value of $\Delta\omega_n$ assumes the minimum value for $n=n_0=\varepsilon[2k/K]$, where $\varepsilon[x]$ is the sign of the integral part of the real number x, and it increases monotonically with removal from the spectral maximum of the number n. In the case where the ratio $2k/K$ is exactly equal to the integer (n_0), q_n vanishes, and from expression (8) we obtain:

$$\langle G_r^c(\omega)\rangle_{n=n_0} \simeq \frac{M}{8K^3}\cdot\frac{K\sigma_\xi\sqrt{\overline{\Omega^2}}T}{2\sqrt{2}\pi^{3/2}}\cdot\frac{K^2\sigma_\xi^2\overline{\Omega^4}S}{g^2}J_{n_0}(2Kh)\left(\frac{\sin\omega_{n_0}T/2}{\omega_{n_0}T/2}\right)^2, \quad (12)$$

where $\omega_{n_0}=\omega-\omega_1-n_0\omega_0$.

Let us use the above-proposed criterion of the presence of sound excitation of the surface in the presence of wind-driven waves. In drawing the distinction with respect to the maximum of the number $n<<n_0$ in the spectrum of the reflected signal in the presence of sound excitation, we arrive at the inequality determining the distinguishable sound frequency $f_0=\omega_0/2\pi$:

$$f_0 > \frac{\sqrt{2}\,\kappa \sigma_\xi \sqrt{\overline{\Omega^2}}}{\pi n}\left[\ln \chi_\nu\left(1-\frac{\alpha_1}{2}\right) - \ln \chi_\nu\left(\frac{\alpha_2}{2}\right) - 2\ln J_n(2\kappa h)\right]^{1/2}. \qquad (13)$$

Inasmuch as the value in the brackets exceeds one, in order to satisfy this inequality the effective width of the "noise" spectrum must be appreciably less than the frequency corresponding to the maximum number n in the presence of sound excitation.

In conclusion, let us present a numerical estimate. Setting $k=2\pi\cdot 10^4 m^{-1}$, $h=10^{-6} + 10^{-6}$ м, $\sigma_\xi=2\cdot 10^{-2}$ м, $\sqrt{\overline{\Omega^2}}=10^{-1}$, $n=1$, $\alpha_1=\alpha_2=0.01$, $\nu=30$, we obtain: $f_0>10$-15 kilohertz.

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UDC 551.465

CALCULATION OF THE AVERAGE WAVE HEIGHT ON THE SURFACE BY DATA ON THE VARIATION IN TIME OF THE WAVE PRESSURE AT DEPTH

By Yu. S. Natkovich, pp 15-17

In this paper a study is made of the method of determining the average wave height on the surface by measurements at a fixed point at known depth of certain parameters of the wave pressure curve that varies in time. The article basically is of a procedural nature and contains the results of a machine experiment on a model, the parameters of which were obtained by the natural observation data. Earlier, a similar "inverse" problem was investigated in [1], but in the "discrete" statement, that is, the wave height was determined with accuracy to a provisional breakdown of the measured wave height range into some number of gradations (classes). This solution can turn out to be unsatisfactory for a problem in which high accuracy of calculating the wave height is required from massive methods. In such cases it is necessary to proceed with the construction of a continuous "inverse" relation.

For the solution of this problem by statistical methods it is necessary to have at our disposal the results of observations (the training sample) and certain assumptions about the hypothetical form of the relation of interest to us. When it is difficult to state propositions of this type, which occurs in the investigated case, the methods in which the approximating functions are not selected a priori but are constructed beginning with the properties of the specific problem (these properties are basic to the training sample) turn out to be effective. One such method was investigated in [2]. In this paper it is used to reproduce the above-mentioned "inverse" relation.[1]

The following are selected as the parameters picked up from the pressure curve [1]: the average period, the dispersion of the periods, the relative width of the spectrum, the average size and power. The estimates of these

[1] The physical arguments forming the basis of the proposed approach remain the same as were presented in [1].

parameters calculated on a set of realizations of the pressure curves are five-dimensional observation vectors which in combination with the values of the average wave heights on the surface "generating" these curves make up the training sample when constructing the approximating function. The approximation procedure is realized on a digital computer by the method discussed in [2] (more precisely, only the second and third steps of the algorithm are used). The results of solving the problem are presented in the form of tables supplied with interpolation rules. For calculation by the tables of the values of the reproduced function (the estimates of the mean wave height) small-scale computational means are used, and in individual cases the calculations can be performed manually.

Now let us consider the results of the digital computer simulation where we limit ourselves for example to the case where the depth of the analysis horizon is equal to 25 meters. At this depth the relative width of the spectrum and the dispersion of the periods become low-informative for determining the wave height; therefore the dependence of the wave height only on the following three parameters is subject to reproduction: the average period, the average size, power. The investigated range of variation of the wave heights is 0.1 to 4.0 meters. The method of forming the training and the control samples is analogous to that described in [1] except that now it is not necessary to break down the range of 0.1 to 4.0 meters into classes. The length of the averaging interval when calculating the estimates of the above-mentioned parameters by the pressure curve is 220 sec (the formulas for calculating them are presented in [1]). The total number of three-dimensional vectors in the sample obtained by digital computer simulation using real statistical data is 8000. Using the random number generator this sample is broken down into the training and control samples (4000 vectors each). The training sample is used to construct a three-dimensional table. For this purpose the range of variation of each parameter (the coordinates) is reduced to a standard; the range of variation of the wave height is also reduced to it. The number of nodes in the table is assumed to be equal to 216 (six nodes with respect to the coordinate). In each of them in accordance with the second step, by the algorithm of [2], the values of the desired function are calculated, that is, the estimates of the average wave heights on the surface.

Comparison of the quality of the approximation using the function given in the table with the quality of the approximation using the in practice widespread approximation of the polynomials of the parameters (usually linear or quadratic) is of interest. For comparison it is convenient to normalize the mean square estimate for the various methods of approximation by the estimate of the wave height dispersion in the control sample. In the table presented below we have the results of the compared experiments.

Item No	Method of approximation	Estimate of the approximate quality
1	Approximation to the mean values	1
2	Linear polynomial	0.8
3	Second-degree polynomial	0.37
4	Table-given function	0.07

As is obvious, the table-given function will permit significant improvement of the quality of the approximation by comparison with the generally accepted approximations using polynomials of the parameters.

The analysis of the estimates of the values of the mean wave height obtained in the control sample using the table-given function demonstrated that the error is on the average about 5% of the true value of the mean wave height.

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UDC 534.21

RELATION OF THE FLUCTUATION SPECTRA OF THE LEVEL AND PHASE OF THE SOUND FIELD TO THE INTERNAL WAVE SPECTRUM

By N. D. Mel'nichuk, pp 18-20

In this report a study is made of the problem of the time spectra of the autocorrelation functions of the level and phase fluctuations of the sound field of a harmonic point source in a waveguide in the presence of internal waves. It was proposed that the average amplitude of the internal wave is equal to zero, and the internal waves are delta correlated both with respect to time and spatial frequencies. The source of the sound field was located at a depth of z_1, and the receiver at a depth of z_2. The spacing between the source and the receiver along the horizontal was x.

The problem is solved by the method of smooth disturbances [1] under the assumption that the process of propagation of the internal waves can be considered quasistationary by comparison with the process of propagation of the sound waves. The sound field was represented in the form of the sum of normal waves [2].

For the spectra of the autocorrelation functions of the level fluctuation $R(\omega)$ and the phase fluctuation $P(\omega)$, the following expressions were obtained:

$$R(\omega) = \sum_j z_j(\omega) E_j(\omega), \tag{1}$$

$$P(\omega) = \sum_j \rho_j(\omega) E_j(\omega), \tag{2}$$

where

$$z_j(\omega) = \alpha_j^2 + \delta_j^2 + (\sigma_j^2 - \delta_j^2)\varphi_j^2(\omega) - \beta_j \psi_j(\omega), \tag{3}$$

$$\rho_j(\omega) = \alpha_j^2 + \sigma_j^2 - (\sigma_j^2 - \delta_j^2)\varphi_j^2(\omega) + \beta_j \psi_j(\omega) \tag{4}$$

[symbol missing] are the coefficients of the relation between the j-th mode of the internal wave and the sound wave, $E_j(\omega)$ is the energy density of the time spectrum of the j-th mode of the internal wave at a frequency ω, and the symbol $\sum_j$ denotes summation of the internal waves with respect to all modes.

In formulas (3), (4) the values of α_j^2, δ_j^2, σ_j^2 and β_j are completely defined by the propagation conditions, that is, hydrology, the given damping for the sound wave modes and also the position of the source and the receiver of the sound waves. The functions $\phi_j(\omega)$ and $\Psi_j(\omega)$ in formulas (3), (4) are the following corresponding expressions:

$$\varphi_j^2(\omega) = \int_0^1 \sin^2\left[\frac{x\varkappa_j^2(\omega)(1-v)v}{2\kappa}\right]dv =$$

$$= \frac{1}{2}\left\{1 - \left[\frac{x\varkappa_j^2(\omega)}{2\pi\kappa}\right]^{-1/2}\left[\cos\left(\frac{x\varkappa_j^2(\omega)}{4\kappa}\right)C\left(\frac{x\varkappa_j^2(\omega)}{2\pi\kappa}\right)^{1/2} + \sin\left(\frac{x\varkappa_j^2(\omega)}{4\kappa}\right)\cdot\right.\right.$$
$$\left.\left.\cdot S\left(\frac{x\varkappa_j^2(\omega)}{2\pi\kappa}\right)^{1/2}\right]\right\},$$

$$\Psi_j(\omega) = \int_0^1 \sin\left[\frac{x\varkappa_j^2(\omega)(1-v)v}{\kappa}\right]dv =$$
$$= \left[\frac{x\varkappa_j^2(\omega)}{2\pi\kappa}\right]^{-1/2}\left[\sin\left(\frac{x\varkappa_j^2(\omega)}{4\kappa}\right)C\left(\frac{x\varkappa_j^2(\omega)}{2\pi\kappa}\right) - \cos\left(\frac{x\varkappa_j^2(\omega)}{4\kappa}\right)S\left(\frac{x\varkappa_j^2(\omega)}{2\pi\kappa}\right)^{1/2}\right]$$

where $\chi_j(\omega)$ is the wave number of the j-th mode of the internal wave of frequency of ω, k is the wave number of the sound wave, and C(x) and S(x) are the Fresnel integrals.

Inasmuch as the process of the propagation of the sound waves in the ocean has a multimodal nature, the resultant sound field is the interference pattern formed by the superposition of individual sound modes. Thus, the level of the sound field depends not only on the levels of the sound modes making up the field, but also on the phases of these modes. Consequently, the correlation of the level fluctuations of the field will depend not only on the correlation of the level fluctuations of the individual modes, but also on the correlation of the phase fluctuations, and also on the mutual correlation of the level and phase fluctuations of these modes. The analogous function is valid also for correlation of the phase fluctuations of the resultant field. All of this has found its reflection in formulas (3) and (4), where with respect to $r_j(\omega)$ the value of $\alpha^2_{\sigma j}+\sigma^2_j\phi_j^2(\omega)$ is the contribution from the correlation of the mode level fluctuations, $\alpha^2_{\delta j}+\delta_j^2[1-\phi_j^2(\omega)]$ from the correlation of the mode phase fluctuations, and $-\beta_j\Psi_j(\omega)$is the contribution from the mutual correlation of the mode phase and level fluctuations; $\alpha^2_j=\alpha^2_{\sigma j}+\alpha^2_{\delta j}$ is an additional term which arises as a result of the different damping of the sound modes.

As for the value of $P_j(\omega)$, the contribution of the correlation of the phase fluctuations in it is $\alpha^2_{\sigma j}+\sigma^2_j[1-\phi^2_j(\omega)]$, and from the correlation of the mode level fluctuations the contribution is equal to $\alpha^2_{\delta j}+\delta^2_j\phi^2_j(\omega)$. The contribution from the mutual correlation of the fluctuations of the levels and phases of these modes, just as with respect to the coefficient $r_j(\omega)$ is equal to $\beta_j\Psi_j(\omega)$, but it has opposite sign.

Thus, it is possible to state that the value of $\theta=d_j^2/\sigma_j^2$ defines the relative contribution of the correlation of the mode phase fluctuations of the sound field to the coupling coefficient for the level and the relative contribution from the correlation of the level fluctuations of these modes to the coupling coefficient for the phase. For $\theta<<1$ this relative contribution is small. For $\theta\approx1$ the correlations of the mode level fluctuations and the correlation of the mode phase fluctuations make an identical contribution to the coupling coefficient. For $\theta>>1$ in the coupling coefficient for the level, the contribution from the correlation of the mode phase fluctuations of the sound field predominates, and in the coupling coefficient for the phase, the contribution from the level fluctuation correlation of these modes predominates.

Let us return to the formulas (1), (2). The coupling coefficients $r_j(\omega)$ and $P_j(\omega)$ are completely defined by the propagation conditions. If we measure the spectra $R(\omega)$ or $P(\omega)$ for different i sound frequencies for constant source and receiver positions and we calculate i values of $r_{ji}(\omega)$ or $P_{ji}(\omega)$ for them, we can put together the following system of equations:

$$R_i(\omega) = \sum_j r_{ji}(\omega) E_j(\omega), \qquad (5)$$

$$P_i(\omega) = \sum_j \rho_{ji}(\omega) E_j(\omega). \qquad (6)$$

Since with an increase in the number j the spectrum $E_j(\omega)$ decreases rapidly [3, 4], in order to solve the systems with respect to $E_j(\omega)$, a small number of linear equations is required. The maximum number i coincides with the maximum number j, the choice of which, in turn, is determined by the required solution precision.

Let us note that the systems can be composed by varying the positions of the receivers and the radiators and also by combining all of the methods.

Thus, using formulas (5) and (6), we solve the inverse problem, that is, by the spectra of the level or phase fluctuations we find the spectrum of the internal waves.

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UDC 534.21

TIME CORRELATION OF NOISE SIGNALS IN THE OCEAN

By V. M. Frolov, pp 21-24

The autocorrelation function of a stationary noise signal propagated in the ocean by any water beam is equal to the Fourier transformation of the spectral power of the received signal

$$K(\bar{z},\tau)=\int_{-\infty}^{\infty}G(\omega,\bar{z})e^{-i\omega\tau}d\omega. \quad (1)$$

In turn, the spectral power of the received signal can be represented in the form:

$$G(\omega,\bar{z})=G(\omega)\cdot\frac{F}{4\pi z^2}10^{-0.1\beta(\omega)R}, \quad (2)$$

where $G(\omega)$ is the power spectrum of the emitted signal considering the frequency characteristic of the receiving system, F is the focusing factor, R is the spacing between the source and the receiver, and $\beta(\omega)$ is the sound damping coefficient.

Substituting (2) in expression (1) and normalizing it, we obtain the autocorrelation coefficient

$$\Gamma(\tau)=\frac{\int_0^{\infty}G(\omega)10^{-0.1\beta(\omega)R}\cos\omega\tau\,d\omega}{\int_0^{\infty}G(\omega)10^{-0.1\beta(\omega)R}d\omega}. \quad (3)$$

It is possible to use formula (3) also to calculate the autocorrelation function of a beam reflected from the wavy sea surface with gently sloping large-scale unevennesses in the case of sufficiently wide-band signal where the time correlation interval of the surface prominences Δt_S is much greater than the correlation interval of the emitted signal Δt_C.

As for the mutual correlation between the two beams, for two water beams it is possible to expect that the mutual correlation function will be determined as before by the formula (3) where the difference of the propagation times with respect to these beams is added to τ inasmuch as the characteristic times of variation of the large-scale unevennesses, the internal waves, and the hydrologic conditions are very large by comparison with the averaging time used to find the correlation. The averaging time is usually the sending time T much greater than Δt_c. When calculating the mutual correlation of the wave beam with a surface beam or two surface beams in view of the high velocity of the surface waves it can turn out that Δt_s is comparable to T. Then formula (3) can only be used in the case of weak wave action when the additional phase signals on reflection of the beams can be neglected. Otherwise, the averaging time must be increased to the point that it becomes much greater than Δt_s. Here the mutual correlation coefficient, for example, of the water beam and the beam reflected from the surface will be equal to

$$\Gamma_s(\tau)=\frac{\int_0^\infty G(\omega)10^{-0,1\beta R}\exp[-2(\frac{\omega}{c}\sigma\sin\Psi)^2]\cos\omega\tau\,d\omega}{\int_0^\infty G(\omega)10^{-0,1\beta R}d\omega}, \quad (4)$$

where c is the speed of sound, σ is the mean square value of the prominences of the surface unevennesses, Ψ is the sliding angle of the mirror beam. If the mutual correlation coefficient of the water beam and the beam experiencing not one but N reflections from the surface is defined, then it is sufficient to introduce the factor N into the exponent (4).

However, in the experiment frequently beams arriving at the reception point at close angles cannot be resolved by the receiving antenna, and then as a result of processing the measurements, an autocorrelation coefficient is obtained not for one beam, but the sum of two beams, but, for example, between one beam and the sum of two others. If the difference in the arrival time Δt of two unresolved beams is large by comparison with Δt_c, the presence of the additional beam leads to the appearance of two separate correlation regions with reduced maximum correlation. However, if Δt is of the order of or less than Δt_c, then the correlation regions are not separated, but overlap. Here the resultant correlation function will be asymmetric, and its maximum value will also be diminished by comparison with the presence of only two beams. The asymmetry and reduction in the maximum mutual correlation coefficient sometimes observed experimentally can also be called this mechanism. For example, in the case where two unresolved beams have amplitudes of I and ε, the autocorrelation coefficient of the sum of these beams and the mutual correlation coefficient of this sum with the third beam will be expressed by the formulas, respectively:

$$\Gamma_1(\tau)=\frac{\int_0^\infty G(\omega)10^{-0,1\beta R}(1+2\varepsilon\cos\omega\Delta t+\varepsilon^2)\cos\omega\tau\,d\omega}{\int_0^\infty G(\omega)10^{-0,1\beta R}(1+2\varepsilon\cos\omega\Delta t+\varepsilon^2)d\omega}, \quad (5)$$

$$\Gamma_2(\tau)=\frac{\int_0^\infty G(\omega)10^{-0,1\beta R}[\cos\omega\tau+\varepsilon\cos\omega(\tau+\Delta t)]d\omega}{\sqrt{\int_0^\infty G(\omega)10^{-0,1\beta R}(1+2\varepsilon\cos\omega\Delta t+\varepsilon^2)d\omega\int_0^\infty G(\omega)10^{-0,1\beta R}d\omega}}. \quad (6)$$

For illustration of the effect of the auxiliary beam on the correlations, a computer was used to calculate an example by formula (5) for a signal with rectangular spectrum in the frequency band of 800-1300 hertz at R=200 km. The damping coefficient is taken in the form of $\beta=0.036\cdot f^{3/2}$ decibels/km, where f is in kilohertz [1]. Fig 1 shows the mutual correlation coefficients for $\varepsilon=0.8$ and $\Delta t=0.5$ milliseconds (the solid lines) and 0.6 milliseconds (the dashed lines). Fig 2 shows the mutual correlation coefficients for $\Delta t=0.5$ milliseconds and $\varepsilon=0.5$ (solid lines) and I (the dashed line).

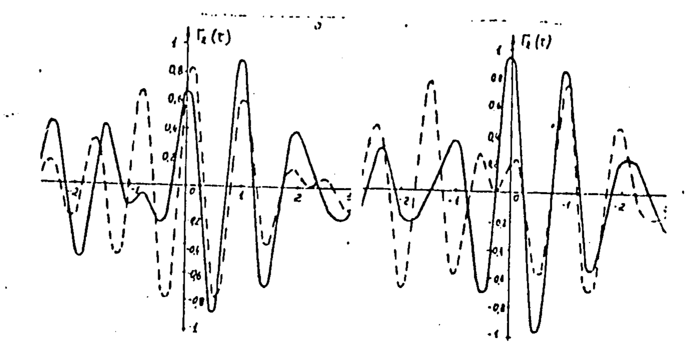

Figure 1 [sic]*

From the figures it is obvious that the correlation coefficients are highly sensitive to the variation in the signal propagation time difference with respect to different beams when this difference is less than Δt_c which in the given case is equal to 2 milliseconds. This indicates that even for wide band emission interference is possible. It is also necessary to note the sensitivity of the correlation coefficients to variation of the amplitudes of the arriving signals.

The fact that the variation of Δt in 0.1 milliseconds corresponding to the variation in the path difference by a total of 15 cm can have a sharp influence on the correlation coefficients, it will lead to the conclusion of the difficulty of predicting the experimental results by measuring the time correlation in the case where there are several unresolved receiving antenna beams with a propagation time difference less than the correlation interval of the emitted signal. In particular, this is connected with the fact that the beam picture is highly sensitive to the small variations of the hydrologic situation.

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*[Translator's note]. Probably the left side of this is Fig 1 and the right is Fig 2.

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SPATIAL CORRELATION FUNCTIONS OF SOME SOURCES OF RANDOM STATIONARY SOUND FIELDS

By M. I. Karnovskiy, I. L. Oboznenko, A. V. Sil'chenko, pp 24-26

Let us define the time-space correlation function of the scattered field on excitation of an arbitrary surface S by a random field of oscillatory velocities v(M, t). It is proposed that in the given coordinate system the Helmholtz equation is separated.

Let $\{v(\vec{r},t\}$ and $\{\Psi(\vec{r}, t)\}$ represent stationary random fields (the set of all possible realizations of the fields) correspondingly for the oscillatory velocity and the velocity potential. It is proposed that the average power of these processes is limited. The solution to the Helmholtz equation

$$(\Delta + K^2)\Psi(\vec{r},\omega) = 0, \quad K = \omega/c \tag{1}$$

under boundary conditions on the surface $\vec{r}=\vec{r}_0 \in s$ for each realization of the velocity potential spectrum

$$\frac{\partial \Psi(\vec{r}_0,\omega)}{\partial n} = -U(\vec{r}_0,\omega) \tag{2}$$

can be represented in the form

$$\Psi(\vec{r},\omega) = \sum_{m,n,\nu} a_{mn\nu}(\omega) X_m(\xi) Y_n(\eta) Z_\nu(\zeta), \tag{3}$$

where X, Y and Z are deterministic functions of the wave coordinates, which are the solution of the second-order ordinary differential equations; $\alpha_{mn\nu}$ is the set of random functions determined from the conditions (2) for each realization.

Let us also represent the realization of the oscillatory velocity on the surface s in the form:

$$U(\vec{r}_0,t) = f_1(\vec{r}_0) f_2(t), \quad f_2(t) = b(t) e^{i\varphi(t)} \tag{4}$$

Here $f_{1,2}$, B and ϕ are random functions. Then the power spectrum with respect to the oscillatory velocity on the surface S will be equal to

$$W(\vec{r}_0,\omega) = \langle |V(\vec{r}_0,\omega)|^2 \rangle , \tag{5}$$

where

$$V(\vec{r}_0,\omega) = f_1(\vec{r}_0) f_2(\omega), \quad f_2(\omega) = \int_{-T}^{T} b(t) e^{i[\varphi(t)-\omega t]} dt, \tag{6}$$

T is the observation time of the realization.

The mutual frequency spectrum of the pressure with respect to space and its correlation functions will be represented according to (3) for two observation points $M_1(\vec{r}_1)$ and $M_2(\vec{r}_2)$ (Fig 1) in the form:

$$g(\vec{r}_1,\vec{r}_2,\omega) = \sum_{m,n,\nu}\sum_{m',n',\nu'} W_{mn\nu}^{m'n'\nu'}(\omega) X_m(\xi_1) X_m^*(\xi_2) Y_n(\eta_1) Y_n^*(\eta_2) Z_\nu(\zeta_1) Z_\nu^*(\zeta_2) \tag{7}$$

$$K(\vec{r}_1,\vec{r}_2,\tau) = \int_{-\infty}^{\infty} g(\vec{r}_1,\vec{r}_2,\omega) e^{i\omega\tau} d\omega , \tag{7'}$$

where

$$W_{mn\nu}^{m'n'\nu'}(\omega) = \int_{-\infty}^{\infty} \langle a_{mn\nu}(\omega) a^*_{m'n'\nu'}(\omega) \rangle d\omega' \tag{7''}$$

is the set of power spectrum components (5) averaged with respect to the realizations and defined from the boundary conditions (2).

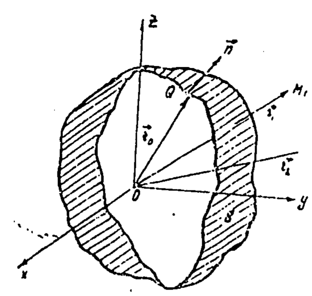

Figure 1

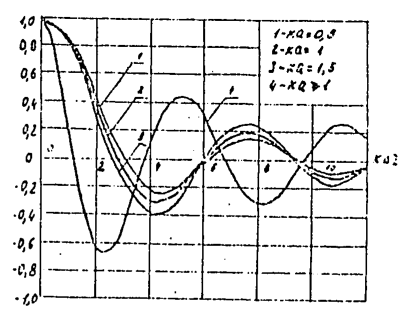

Figure 2

In order to define the functions (7) and (7') it is necessary to give the specific form of the source surface and the correlation (spectral) properties of the exciting functions (4) or (5).

As an example let us consider the spherical source of radius $|\vec{r}_0|=a$. In this case for each realization instead of (3) we have:

$$\Psi(\vec{z},\omega)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n} a_{mn}(\omega)h_n(\varepsilon)P_n^m(x)e^{im\varphi}, \qquad (8)$$

where

$$a_{mn}(\omega)=C_{mn}(\omega)\int_0^{2\pi}\int_{-1}^{1} f_1(x)P_n^m(x)e^{im\varphi}d\varphi dx, \qquad (9)$$

$$x=\cos\theta,\ \varepsilon=kz,\ C_{mn}(\omega)=-\frac{2n+1}{2k}\cdot\frac{(n-m)!}{(n+m)!}\cdot\frac{f_2(\omega)}{h'_n(\varepsilon_0)}.$$

Here $h_n(\varepsilon)$ and $P_n^m(x)$ are the first type spherical Henkel function and the associated Legendre functions respectively, and the stroke indicates differentiation with respect to the argument.

If we propose that the sources on the surface s, δ are correlated, $f_1(x_1)\cdot f_1(x_2)=\delta(x_1-x_2)$ then the mutual spatial spectrum according to (7) and (8) will be:

$$g(\vec{z}_1,\vec{z}_2,\omega)=\frac{1}{2k^2}\langle|f_2(\omega)|^2\rangle\sum_{n=0}^{\infty}\sum_{m=-n}^{n}(2n+1)\frac{(n-m)!}{(n+m)!}D_{mn}(\vec{z}_1,\omega)D_{mn}(\vec{z}_2,\omega), \qquad (10)$$

where

$$D_{mn}(\vec{z},\omega)=-\frac{h_n(\varepsilon)P_n^m(x)e^{im\varphi}}{h'_n(\varepsilon_0)}.$$

Setting $\theta_1=\theta_2=0,\ \varepsilon_2=kz_2\gg m+1,\ \varepsilon_1=k(z_2+\Delta z)$ the normalized space-time correlation assumes the form

$$R(\Delta z,\tau)=\sum_{m=0}^{\infty}\int_0^{\infty}\frac{\langle|f_2(\omega)|^2\rangle(2n+1)\cos\omega\alpha\, d\omega}{\omega^4/|h'_n(\varepsilon_0)|^2}\Bigg/\sum_{m=0}^{\infty}\int_0^{\infty}\frac{\langle|f_2(\omega)|^2\rangle(2n+1)}{\omega^4/|h'_n(\varepsilon_0)|^2}d\omega, \qquad (11)$$

where $\alpha=\Delta r/c-\tau$ defines the time-space shift of the observation points M_1 and M_2. Formula (11) is valid in the far field for any wave dimensions of $\varepsilon_0=ka$ of the source. In the region of small wave dimensions, formula (11) is significantly simplified:

$$R(\Delta z,\tau)=\frac{\int_0^{\infty}\langle|f_2(\omega)|^2\rangle\cos\omega\alpha\,[1+\frac{3}{7}\varepsilon_0^2+\frac{5}{81}\varepsilon_0^4+\dots]d\omega}{\int_0^{\infty}\langle|f_2(\omega)|^2\rangle[1+\frac{3}{7}\varepsilon_0^2+\frac{5}{81}\varepsilon_0^4+\dots]d\omega}. \qquad (12)$$

Fig 2 shows the calculated values of the function (12) for $\tau=0$ and $\langle|f_2(\omega)|^2\rangle=1$, $[0,\omega]$, for certain values of the wave dimension of the source ε_0 and also the value of the correlation function for $\varepsilon_0 >> 1$ which for $<|f_2(\omega)|^2> = 1/\omega^2$, $[\omega, \infty]$ is equal to:

$$R(\Delta z, \tau) = \left\{ \frac{\cos K\Delta z}{(K\Delta z)^3} - \frac{\sin K\Delta z}{2(K\Delta z)^2} - \frac{\cos K\Delta z}{2K\Delta z} + \frac{1}{2}\left[\frac{\pi}{2} - Si(K\Delta z)\right] \right\} (K\Delta z)^3, \quad (13)$$

Si(x) is the integral sine.

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PROBLEM OF THE FORMATION OF THE WAVE FRONT IN A PLANE-PARALLEL WAVEGUIDE

By V. I. Genis, I. L. Oboznenko, L. Ya. Taradanov, pp 27-31

The representation of the field in the plane-parallel waveguide based on expansion of [1] presupposes the selection of the number of expansion terms on the order of $n \geq kH/\pi$, where H is the height of the waveguide. Some results with respect to this problem were obtained in [2] where the results are presented from calculating the wave front of the point source in the plane-parallel waveguide, the upper boundary of which is absolutely soft, and the lower is absolutely hard. In this paper it is demonstrated that if for the field calculations for the indicated waveguide we take into account the terms of the expansion, taken by the groups with respect to $n<kH/\pi$, in the various sections of the range of mode numbers $i[1, kH/\pi]$, then as a result of the summation with respect to the selected regions, we obtain the results in each of the selected region of values i differing from those which correspond to calculation of the field for $n=kH/\pi$. In Fig 1 (curves 1, 2) and Fig 2 we have the results of calculating the field amplitude according to formulas [2] where the number of summed modes was selected equal to $(kH/4\pi)(H/2\lambda)$ in different sections of the range $i[1, kH/\pi]$. From the results obtained it is obvious that none of the presented graphs corresponds to the real one (graph 3, Fig 3) obtained for the values of $i[1, kH/\pi]$, where the results are also presented from calculating the wave front amplitude for $i[1, kH/2\pi]$ (graph 1) and for $i[kH/2\pi+1, kH/\pi]$ (graph 2). As is obvious from Fig 3, the data from calculating the field both in the former and in the latter case do not give a complete representation of the field structure at the same time as the graphical sum of the curve 1, 2 in Fig 3 corresponds quite precisely to the amplitude distribution of the wave front. From what has been stated above it follows that the reduction of the number of summed normal waves of any order is not admissible.

Let us then consider the waveguides with identical absolutely reflecting boundaries. According to [1], the field in the plane-parallel waveguide with two absolutely rigid boundaries will be described by the expression:

$$\Phi(z,x) = -\frac{jQ}{2H}\left\{\frac{H_0^{(2)}(Kz)}{2} + \sum_{i=1}^{\infty} \cos\frac{i\pi}{H}x_0 H_0^{(2)}(\gamma_i z)\cos\frac{i\pi}{H}x\right\}, \quad (1)$$

where q is the source duration; $H_0^{(2)}$ is the cylindrical Henkel function; H, L_0, r are presented in [2]; $\gamma_i = k\sqrt{1-(\frac{i\pi}{kH})^2}$, $k = \frac{2\pi}{\lambda}$.

Using the asymptotics of the Henkel functions for $\gamma_i r \gg 1$, for the field in the far zone we obtain:

$$\Phi(z,x) = -\frac{jq}{2H}\left\{\frac{1}{2}\frac{\sin(kz-\frac{\pi}{4})}{(kz)^{1/2}} + \sum_{i=1}^{kH/\pi}\cos\left(\frac{i\pi}{H}x_0\right)\cos\left(\frac{i\pi}{H}x\right) \times \frac{\sin(\gamma_i z-\frac{\pi}{4})}{(\gamma_i z)^{1/2}} + j\left[\frac{1}{2}\frac{\cos(kz-\frac{\pi}{4})}{(kz)^{1/2}} + \sum_{i=1}^{kH/\pi}\cos\left(\frac{i\pi}{H}x_0\right)\cos\left(\frac{i\pi}{H}x\right)\frac{\cos(\gamma_i z-\frac{\pi}{4})}{(\gamma_i z)^{1/2}}\right]\right\} \quad (2)$$

Performing the analogous transformations, we find the expressions describing the field in the waveguide with two absolutely soft boundaries

$$\Phi(z,x) = \sum_{i=1}^{kH/\pi}\sin\left(\frac{i\pi}{H}x_0\right)\sin\left(\frac{i\pi}{H}x\right)\frac{\sin(\gamma_i z-\frac{\pi}{4})}{(\gamma_i z)^{1/2}} + j\sum_{i=1}^{kH/\pi}\sin\left(\frac{i\pi}{H}x_0\right)\sin\left(\frac{i\pi}{H}x\right)\frac{\cos(\gamma_i z-\frac{\pi}{4})}{(\gamma_i z)^{1/2}} \quad (3)$$

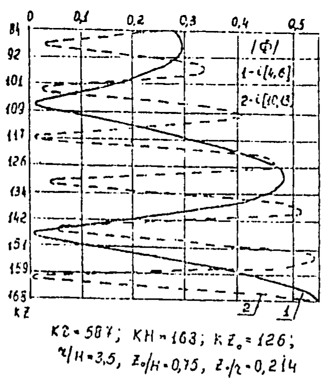

Figure 1

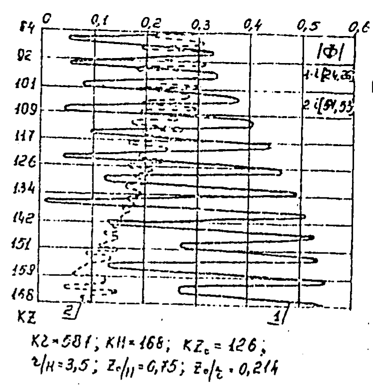

Figure 2

The results of calculating the modulus of the velocity potential according to expressions (2) and (3) presented in Figures 4, 5 demonstrated that in contrast to the waveguide with different absolutely reflecting boundaries in the waveguides with identical reflecting boundaries focusing of the field along the axis of the point source is not observed. In this case the field is distributed more uniformly with respect to depth of the waveguide. The amplitude oscillations of the wave front are caused by the presence of modes of higher orders. The results of calculating the amplitude on the wave front at differences from the radiator for the waveguide with two absolutely rigid boundaries (if the number of summed normal waves is selected equal to (kH/π) (2H/π), are presented in Fig 4, a. It must be noted that with an increase in the distance from the radiator the oscillations diminish, and the effect of the higher-order modes is attenuated, which is obvious from Fig 4a (curves 1, 2, 3). The calculations were performed for a different number of summed modes, in particular, the results of the calculations are presented where the number of terms was taken equal to kH/2π; kH/4π; kH/8π; kH/16π (Fig 4 b,c). Analyzing these graphs, it is possible to draw the conclusion that at the distances r⩾H the decrease in the number of summed normal waves by 2 times, just as in the case of the waveguide, investigated in [2], leads to entirely invalid results. In Fig 4, b curves 1,2,3,4 correspond to the number of summed normal waves kH/2π; kH/4π; kH/8π;kH/16π. The graphs depicted in Fig 4 correspond to values of H=168 and kz_0=125.

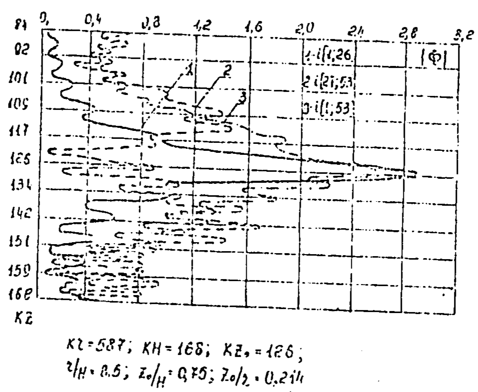

Figure 3

In Figures 5a, b the results are presented from calculating the wave front amplitude in the waveguide with two absolutely rigid boundaries for the same values of kr and $k\alpha_0$ as in Figures 4a, b and c, but for values of kH=419. Comparing the results presented in Figures 4a and 5a, it is possible to note that the oscillations increase with an increase in kH for all other equal parameters This is explained by the fact that a larger number of normal waves participate in the formation of the field.

Actually, the field in the waveguide can be represented in the form of the sum of the waves propagated in accordance with the distribution of the emitted energy with respect to the beams formed as a result of reflection from the waveguide boundaries.

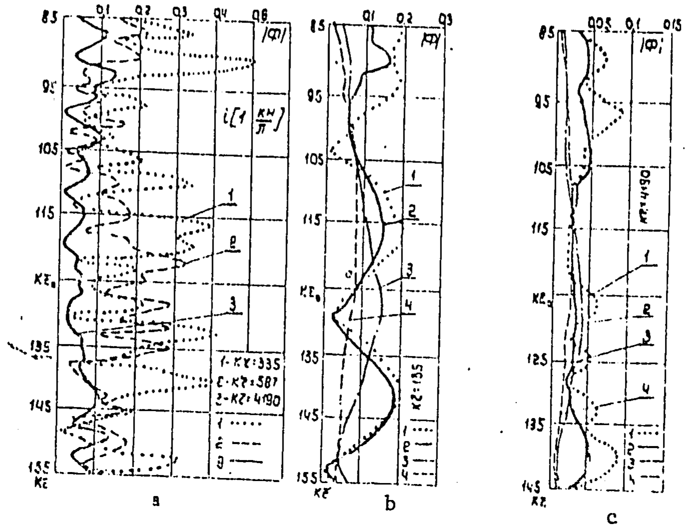

Figure 4

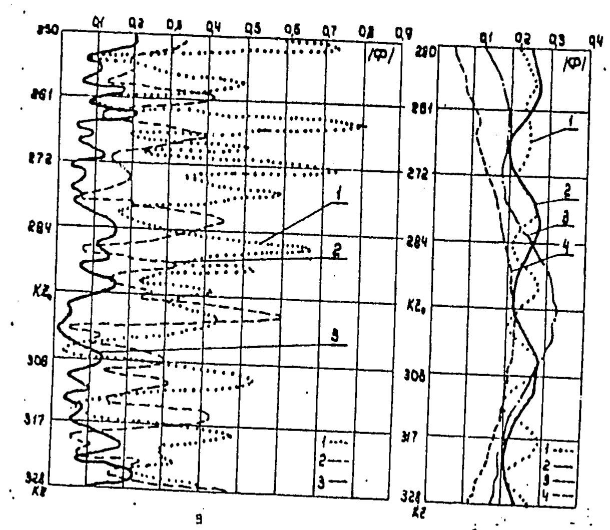

Figure 5

Using the vector addition rule, it is possible to show that depending on kH, the number of projections of the vectors on the axis of the source making a perceptible contribution to the formation of the field in the waveguide will be different and greater, the greater kH.

The focusing of the field with respect to the source axis in the waveguide is explained by the fact that the vector corresponding to the beam with respect to the source axis and the vector corresponding to the beams located at small angles with respect to the source axis have prevailing influence on the field formation in the waveguide, especially in the near zone.

As the distance increases from the source, the focusing of the field weakens, the number of beams not reflected from the boundaries decreases, and the field formation in the waveguide takes place as a result of a larger and larger number of "beams" reflected from the waveguide boundaries.

The second cause of attenuation of the focusing of the field with respect to the source axis in the waveguide is explained by the fact that the beams emerging from the source at large angles to the waveguide boundaries travel a significantly greater distance and, in addition, the intensity of the multiply rereflected beams decreases as a result of splitting of the beam on reflection from the boundary.

The results of the calculations indicate that the structure of the field of the emitted wave in the waveguides with absolutely reflecting boundaries will be identical for rigid and soft boundaries.

For the final conclusion regarding the effect of the waveguide boundaries on the field formation, experimental studies are needed which will permit estimation not only of the effect of the boundaries on the field structure in the waveguides but also simultaneous estimation of the energy losses of the emitted wave on propagation in layered nonuniform media.

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SPECTRA OF INTENSIVE HYDROACOUSTIC WIDE BAND SIGNALS

By S. M. Gorskiy, Ye. N. Pelinovskiy, Yu. V. Petukhov, V. Ye. Fridman, pp 32-33

A discussion is presented below of the results of investigating the processes of transformation of the spectrum of an intensive wide band signal excited by an underwater explosive source.

As is known, within the framework of the method of nonlinear geometric acoustics, the effect of stratification and also random nonuniformity is taken into account by the integral expression so that in the presented coordinates the environment can be considered uniform, and the wave planar [1]. Let us consider the pulse of exponential shape given at some distance $r=R_0$ from the source of the explosion. The propagation of such a pulse is described by the solution in the form of a Rieman wave with the corresponding boundary conditions at the discontinuity. It is possible to demonstrate that the amplitude of such a pulse $\mathcal{P}_s$ and its characteristic duration θ_s varies with distance as follows:

$$\mathcal{P}_s = \frac{\sqrt{1+2x}-1}{x}, \qquad \theta_s = \sqrt{1+2x} - \ln\frac{1}{2}(1+\sqrt{1+2x}), \tag{1}$$

where $x=z/R_*$, z is the reduced coordinate, $\mathcal{P}$ is the reduced pressure, expressed by the following relations:

$$\mathcal{P} = p\left\{\frac{\Delta\cdot C_0\cdot\rho_0}{\Delta_0\cdot C\cdot\rho}\right\}^{1/2}, \qquad z = \int_{\ell_0}^{\ell}\left\{\frac{\Delta_0\cdot\rho_0\cdot C_0^5}{\Delta\cdot\rho\cdot C^5}\right\}^{1/2} d\ell. \tag{2}$$

Here the area of the radiation tube Δ and the characteristic nonlinearity scale ($R_*=2C_0^5\rho_0 T_m/(\gamma+1)P_m$) are also introduced; T_m and P_m are the initial pulse parameters for which the expressions (1) are normalized; C_0 and ρ_0 are the parameters of the environment near the source $r=R_0$);

γ is the adiabatic constant. In the presented notation the Rieman solution has the following form:

$$\mathcal{P} = exp\{-(\tau + x\mathcal{P})\}, \tag{3}$$

where $\tau=t/T_m$. Now it is possible to write the expression for the spectrum of this pulse at an arbitrary distance:

$$B(\omega, x) = \frac{1}{2\pi}\int_{-(\theta_s-1)}^{\infty} exp\{-(\tau + x\mathcal{P})\} e^{-i\omega\tau} d\tau \tag{4}$$

where ω is the dimensionless frequency normalized for T_m.

Making the transition in (4) to integration along the characteristic, we obtain:

$$B(\omega, x) = \frac{1}{2\pi}\int_{\xi_s}^{\infty} exp\{-(i\omega+1)\xi + i\omega x e^{-\xi}\}(1 + xe^{-\xi}) d\xi. \tag{5}$$

Here the lower integration limit $\xi_s = ln\frac{1}{2}(1+\sqrt{1+2x})$ is defined by the variation in the duration of the direct pulse on propagation.

The calculations on the computer of the expression for $|B(\omega,x)|^2$ demonstrated that there is no buildup of the low frequencies (see Fig 1). All the

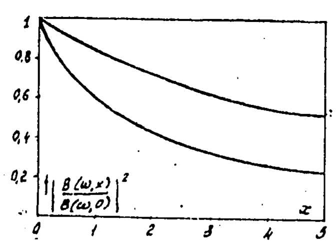

Figure 1

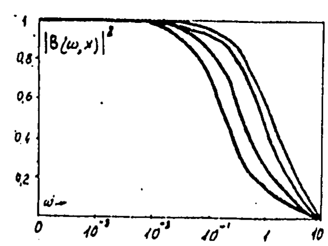

Figure 2

frequencies damp. The higher the frequency, the more intensely the spectral component corresponding to this frequency damps depending on the distance traveled by the wave (see Fig 2). The results obtained make it possible to draw the following conclusions. Inasmuch as the nonlinearity is of a cumulative nature, for calculation of the spectra of the intense hydro-acoustic signals it is necessary, along with variation of the signal parameters as a result of nonlinearity, to consider the nonlinear distortion of the shape of the profile of such a signal. Failure to consider this phenomenon leads to results (see reference 2) which do not agree with the experimental data.

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ADDITIVE MODELS OF SONAR SIGNALS

By V. V. Ol'shevskiy, pp 33-40

1. The increasing scales of the use of the acoustic methods when investigating various hydrophysical characteristics of the World Ocean [1] lead to the necessity for a more detailed investigation and improvement of the probability models of the observed hydroacoustic processes and fields. One of the important areas in this field is the investigation of reflecting and dispersing properties of the nonuniformities of an aqueous environment and its boundaries by the active sonar techniques [1-7]. It is sufficient to state that which probability model of the reflected and the scattered signal is adopted in this case essentially determines the interpretation of the theoretical and experimental results obtained, and, in the final analysis, the quantitative measure of the estimate of the errors in the investigation and scientific reliability of these results as a whole [2, 5, 8].

At the present time several probability models of the sonar signals have been developed [2-7], each of which pertains to defined types of propagation, reflection and dispersion of the sound waves in an aqueous environment, at its nonuniformities and boundaries. Among these models the primary role is played by the so-called active models in accordance with which the sonar signal is represented in the form of the sum of the coherent and random components. A similar representation, on the one hand, highly constructive, permits us to obtain many useful results, to solve a broad class of measuring (systems) problems on the basis of this model; on the other hand, far from always (see, for example, [5, 6]) will the physical interpretation of the various conditions of propagation of the sound waves permit sufficiently simple determination of the additive equivalent of the models of a more complex type (multiplicative models, models of the canonical, multibeam and other types). Nevertheless, the additive models of the sonar signals are of great interest, and their theory must unconditionally be developed, the more so in that the probability models must be developed directionally, in accordance with the circuits which are formed and more precisely defined each time.

2. Let us initially consider a general scheme for the formation of the sonar signals which will permit correct formulation of the restrictions

which are imposed on the conditions of the possibility of using additive models of echoes in active sonar.

Under sufficiently general assumptions [2, 6] the echo $Z_s(t)$ can be represented in the form of the following operator model:

$$Z_s(t) = A_П M_2 T M_1 A_И \{Z_c(t)\}, \qquad (1)$$

where $Z_c(t)$ is the emitted signal; $A_И$, $A_П$ are the operators of the radiating and receiving antennas; M_1 and M_2 are the operators of the propagation of the acoustic signal from the radiating antenna to the object of location and from the object to the receiving antenna respectively; T is the operator of the object of location (by $Z_c(t)$ and $Z_s(t)$ we mean the complex envelopes of the corresponding signals expressed in terms of their quadrature components [2, 5]).

In accordance with the additive model the echo $Z_s(t)$ is represented in the form of the sum of two coordinates: coherent $Z_s^{(k)}(t)$ and random $Z_s^{(c)}(t)$, that is:

$$Z_s(t) = Z_s^{(k)}(t) + Z_s^{(c)}(t). \qquad (2)$$

Our mission is to obtain mathematical representations for the characteristic components $Z_s^{(k)}(t)$ and $Z_s^{(c)}(t)$ of the resultant echo $Z_s(t)$ in terms of the corresponding characteristics of the signals described according to the operator model (1) by different operators M_1, T and M_2. We shall consider that each of these operators is additively "split" into two -- coherent and random -- so that the formation of the echo takes place by the scheme presented in Fig 1.

Here:

$$M_1^{(k)} V M_1^{(c)} = M_1; \quad T^{(k)} V T^{(c)} = T; \quad M_2^{(k)} V M_2^{(c)} = M_2. \qquad (3)$$

According to the presented scheme for the formation of the echo, its coherent component $Z_s^{(k)}(t)$ is found as follows:

$$Z_s^{(k)}(t) = A_П M_2^{(k)} T^{(k)} M_1^{(k)} A_И \{Z_c(t)\}. \qquad (4)$$

The random component according to Fig 1 of the echo $Z_s^{(c)}(t)$ is formed as the sum of seven components:

$$\begin{aligned} Z_s^{(c)}(t) = {} & A_П M_2^{(c)} T^{(k)} M_1^{(k)} \{Z_c(t)\} + \\ & + A_П M_2^{(k)} T^{(c)} M_1^{(k)} A_И \{Z_c(t)\} + A_П M_2^{(c)} T^{(c)} M_1^{(k)} A_И \{Z_c(t)\} + \\ & + A_П M_2^{(k)} T^{(k)} M_1^{(c)} A_П \{Z_c(t)\} + A_П M_2^{(c)} T^{(k)} M_1^{(c)} A_И \{Z_c(t)\} + \\ & + A_П M_2^{(k)} T^{(k)} M_1^{(c)} A_П \{Z_c(t)\} + A_П M_2^{(k)} T^{(c)} M_1^{(c)} A_И \{Z_c(t)\}. \end{aligned} \qquad (5)$$

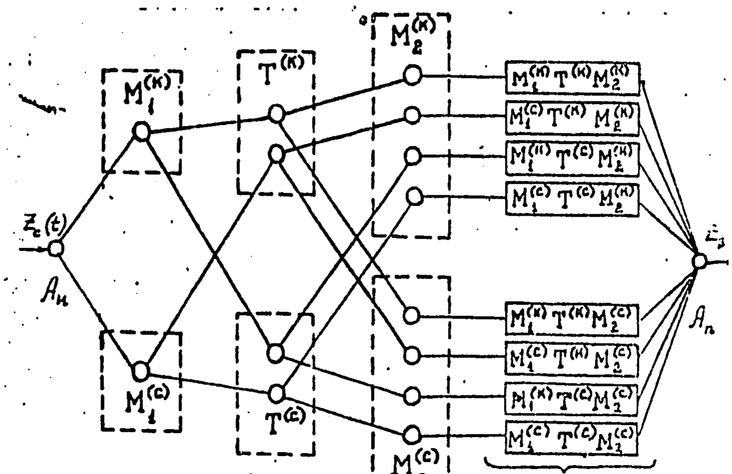

Figure 1. Diagram of the shaping of the echo based on the additive model

Key:
1. Interpretation of the type of signal component

Thus, in accordance with the adopted model (3) which describes the properties of the operators M_1, T and M_2 and considering expressions (4) and (5), the coherent component of the echo $Z_S^{(k)}(t)$ is formed only in the case of simultaneous coincidence of the coherent component of the operators M_1, T and M_2, and the random component of the echo $Z_S^{(c)}(t)$ includes all the combinations of the components of the operators M_1, T and M_2 under the condition that at least one of them is represented by a random component.

3. Now let us consider the energy chacteristics of the echoes corresponding to the additive model. The average total intensity J_S of the echo can be determined using the following expression:

$$J_S = \frac{P_A \gamma_И A_{M_1} S_T A_{M_2}}{(4\pi)^2 \rho_1^2 \rho_2^2} 10^{-0.1\beta(\rho_1+\rho_2)}, \qquad (6)$$

where P_A is the acoustic emitted power; $\gamma_И$ is the coefficient of the axial concentration of the radiating antenna; ρ_1 and ρ_2 are the distances from the radiating antenna to the object of location and from the object to the receiving antenna respectively; A_{M_1} and A_{M_2} are the propagation anomlies corresponding to the effect of the operators M_1 and M_2; S_T is the scattering cross section of the object of location taking into account the effect of the operator T; β is the absorption coefficient. Beginning with the two-component additive models (3) of the operators M_1, T and M_2, we can write that

$$A_{M_1} = A_{M_1}^{(K)} + A_{M_1}^{(C)}, \quad S_T = S_T^{(K)} + S_T^{(C)}, \quad A_{M_2} = A_{M_2}^{(K)} + A_{M_2}^{(C)}, \tag{7}$$

where $A_{M_1}^{(k)}$, $S_T^{(k)}$ and $A_{M_2}^{(k)}$ characterize coherent component of the operator A_{M_1}, S_T and A_{M_2}, and $A_{M_1}^{(c)}$, $S_T^{(c)}$ and $A_{M_2}^{(c)}$ are the random components of these operators.

The possibilities and admissibilities of the representations of (7) must, of course, be substantiated. In addition, it follows to keep in mind that:

$$A_{M_1} = A_{M_1}(A_И, A_П), \quad S_T = S_T(A_И, A_П), \quad A_{M_2} = A_{M_2}(A_И, A_П),$$

that is, that these operators generally speaking depend on the characteristics of the radiating and receiving antennas (see, for example [9]).

Now let us introduce the notation:

$$q_{M_1}^{(K)} = \frac{A_{M_1}^{(K)}}{A_{M_1}}, \quad q_{M_1}^{(C)} = \frac{A_{M_1}^{(C)}}{A_{M_1}}, \quad q_T^{(K)} = \frac{S_T^{(K)}}{S_T}, \quad q_T^{(C)} = \frac{S_T^{(C)}}{S_T}, \quad q_{M_2}^{(K)} = \frac{A_{M_2}^{(K)}}{A_{M_2}}, \quad q_{M_2}^{(C)} = \frac{A_{M_2}^{(C)}}{A_{M_2}}. \tag{8}$$

In accordance with the investigated model

$$J_S = J_S^{(K)} + J_S^{(C)}, \tag{9}$$

where $J_s^{(k)}$ and $J_s^{(c)}$ are the intensities of the echo describing its coherent and random components, respectively. Then using relations (6)-(9) and also considering the schematic in Fig 1 and the representations (4) and (5), for $J_s^{(k)}$ and $J_s^{(c)}$, we obtain:

$$J_S^{(K)} = \frac{P_A \gamma_И A_{M_1} S_T A_{M_2}}{(4\pi)^2 \rho_1^2 \rho_2^2} q_{M_1}^{(K)} q_T^{(K)} q_{M_2}^{(K)} 10^{-0.1\beta(\rho_1+\rho_2)}, \tag{10}$$

$$\begin{aligned} J_S^{(C)} = \frac{P_A \gamma_И A_{M_1} S_T A_{M_2}}{(4\pi)^2 \rho_1^2 \rho_2^2} \big(& q_{M_1}^{(K)} q_T^{(K)} q_{M_2}^{(C)} + q_{M_1}^{(K)} q_T^{(C)} q_{M_2}^{(K)} + \\ & + q_{M_1}^{(K)} q_T^{(C)} q_{M_2}^{(K)} + q_{M_1}^{(C)} q_T^{(K)} q_{M_2}^{(K)} + q_{M_1}^{(C)} q_T^{(K)} q_{M_2}^{(C)} + \\ & + q_{M_1}^{(C)} q_T^{(C)} q_{M_2}^{(K)} + q_{M_1}^{(C)} q_T^{(C)} q_{M_2}^{(C)} \big) 10^{-0.1\beta(\rho_1+\rho_2)}. \end{aligned} \tag{11}$$

Now if we consider that from (8) we have the equalities:

$$q_{M_1}^{(K)} + q_{M_1}^{(C)} = 1, \quad q_T^{(K)} + q_T^{(C)} = 1, \quad q_{M_2}^{(K)} + q_{M_2}^{(C)} = 1, \tag{12}$$

then (11) can be reduced to the following, simpler expression:

$$J_S^{(c)} = \frac{\rho_a \gamma_H A_{M_1} S_T A_{M_2}}{(4\pi)^2 \rho_1^2 \rho_2^2} (1 - q_{M_1}^{(k)} q_{M_2}^{(k)} q_T^{(k)}) 10^{-0.1\beta(\rho_1+\rho_2)}. \qquad (13)$$

Let us note that (13) can be obtained immediately on the basis of formulas (6), (9) and (10), but the awkward conclusion derived by us illustrates in more detail the pattern of formation of the additive model of the echo.

4. Let us establish some useful representations for the intensities $J_s^{(k)}$ and $J_s^{(c)}$ of the coherent and random components of the echo in terms of the mutual correlation coefficients of the sonar signals corresponding to the effect of the operators M_1, T and M_2. The fact is that when performing theoretical and experimental studies (see, for example, [1-6]) primarily a study is made of the energy and correlation characteristics, primarily, the direct acoustic signals (the effect of the operators M_1 and M_2) and secondly, the echoes from different objects under uniform standard propagation conditions (the effect of the operator T). This means that it would be desirable to relate the parameters $q_{M_1}^{(k)}$, $q_{M_2}^{(k)}$ and $q_T^{(k)}$ characterizing the relative levels of the coherent components caused by the separate effect of the operators M_1, T and M_2 with the corresponding correlation coefficients R_{M_1}, R_T and R_{M_2} describing the distortions of the signals also as a result of separate effect of the indicated operators.

The discussed relation can be obtained on the basis of the following relations. Let $Z_1(t)$ be the initial signal, and let $L=(M_1 \wedge T \wedge M_2)$ be the two-component additive operator, $L=L^{(k)}+L^{(c)}$ generating the signal:

$$Z_L(t) = \sqrt{J_L^{(k)}}\, Z_1(t) + \sqrt{J_L^{(c)}}\, Z(t), \qquad (14)$$

where $Z_L(t)$ is the resultant signal obtained from the effect of the operator L; $Z_1(t)$ and $Z(t)$ are the coherent and random components of the signal $Z_L(t)$, the intensities of which are equal to $J_L^{(k)}$ and $J_L^{(c)}$, respectively, so that

$$\langle |Z_1(t)|^2 \rangle = 1, \quad \langle |Z(t)|^2 \rangle = 1, \qquad (15)$$

$$J_L = J_L^{(k)} + J_L^{(c)}, \qquad (16)$$

where J_L is the average intensity of the signal $Z_L(t)$.

The mutual correlation coefficient

$$R_L = \frac{\langle Z_1(t) Z_L^*(t) \rangle}{\sqrt{J_L}} \qquad (17)$$

will characterize the relation of the initial signal $Z_1(t)$ and the resultant signal $Z_L(t)$. Substituting (14) in (17) and considering (15) and (16), we find that:

$$R_L = \sqrt{q_L^{(K)}}, \qquad (18)$$

where

$$q_L^{(K)} = \frac{J_L^{(K)}}{J_L} \qquad (19)$$

is the ratio of the average intensity of the coherent component of the two-component additive signal to its overall intensity.

It is easy to interpret expression (18) as applied to the operators $L=M_1$, $L=T$ and $L=M_2$ so that:

$$q_{M_1}^{(K)} = R_{M_1}^2, \quad q_T^{(K)} = R_T^2, \quad q_{M_2}^{(K)} = R_{M_2}^2, \qquad (20)$$

where R_{M_1}, R_T and R_{M_2} are the mutual correlation coefficients of the signal defined by the separate effect of the operators M_1, T and M_2, respectively.

Now considering the representations of (20) it is possible to rewrite expressions (10) and (13) in the following form:

$$J_S^{(K)} = \frac{P_A \gamma_H A_{M_1} S_T A_{M_2}}{(4\pi)^2 \rho_1^2 \rho_2^2} R_{M_1}^2 R_{M_2}^2 R_T^2 \, 10^{-0,1\beta(\rho_1+\rho_2)}, \qquad (21)$$

$$J_S^{(C)} = \frac{P_A \gamma_H A_{M_1} S_T A_{M_2}}{(4\pi)^2 \rho_1^2 \rho_2^2} (1 - R_{M_1}^2 R_T^2 R_{M_2}^2) 10^{-0,1\beta(\rho_1+\rho_2)} \qquad (22)$$

5. An important element of the development of the additive models of the sonar signals is the establishment of the correspondence of the characteristics of the real signals and their additive equivalents. In order to solve this problem it is necessary first of all to determine the criterion of correspondence of the signal characteristics and its additive equivalent and, secondly, to find the quantitative expression by means of which the values of the mutual correlation coefficients R_{M_1}, R_T and R_{M_2} can be defined specifically. It must also be noted that the investigation of the possibilities of the additive representation of the propagation anomalies and the dispersion cross sections in the form of (7) and the matching of these representations to the results of the investigations in the multibeam reverberation area (see, for example [9]) is of great interest. These problems, which are interesting in scientific respects and have practical value when investigating the ocean by the sonar methods, require, however, the performance of a special study, and they go beyond the framework of this paper.

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SOME RESULTS OF EXPERIMENTAL STUDIES OF REVERBERATION USING WIDE-BAND EMISSION

By A. L. Gonopol'skiy, Ya. I. Gronskiy, T. V. Polyanskaya, pp 40-43

The use of wide-band sources, including explosive signals, is basically connected with measuring the local scattering coefficients. The scattering characteristics are obtained by processing the signals with the help of octave or 1/3 octave filters. However, the solution of some of the applied problems in the case of using wide-band emission is connected with detection of a narrow-band signal against a background of intensive reverberation interference, which causes the necessity for studying the distribution law of the fluctuations of the reverberation spectral components at the output of the filters matched with respect to band with the useful signal.

If in the investigated case we use a discrete canonical model of reverberation developed in [1], then for the explosive signal which is described by the time function as follows:

$$p(t) = A_0 e^{-\beta t} \sigma^{\delta}(t),$$

where β is the pulse damping coefficient $\sigma^{\delta}(t)$ is the single discontinuity function, it is possible to obtain expressions for the dispersion σ_F^2, the correlation function $B_F(\tau)$ and the correlation coefficient $R_F(\tau)$:

$$\sigma_F^2 = \langle n_1 \rangle \langle a^2 \rangle \int_{-\infty}^{\infty} A_0 e^{-2\beta t} \sigma^{\delta}(t) dt = \langle n_1 \rangle \langle a^2 \rangle \frac{A_0^2}{2\beta}, \quad (1)$$

$$B_F(\tau) = \langle n_1 \rangle \langle a^2 \rangle \int_{-\infty}^{\infty} A_0^2 e^{-\beta t} e^{-\beta(t+\tau)} \sigma^{\delta}(t) \sigma^{\delta}(t+\tau) dt = \sigma_F^2 e^{-\beta|\tau|}, \quad (2)$$

$$R_F(\tau) = e^{-\beta|\tau|} \quad (3)$$

The power spectrum will be found as a Fourier transformation of an autocorrelation function:

$$G(\omega) = \int_{-\infty}^{\infty} B_F(\tau) e^{-j\omega\tau} d\tau = \int_{-\infty}^{\infty} \sigma_F^2 e^{-\beta|\tau|} e^{-j\omega\tau} d\tau = \frac{2\beta\sigma_F^2}{\beta^2+\omega^2}. \quad (4)$$

For the explosion signal used by us β assumes values on the order of 10^4 1/sec; therefore the power spectrum will have a sufficiently extended section with uniform average density. The average power of the process is defined as:

$$W = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\beta\sigma_F^2}{\beta^2+\omega^2}d\omega = \sigma_F^2. \qquad (5)$$

The obtained equality of the process dispersion to its average power is characteristic of random processes with different distribution laws, including for normally distributed processes. In addition, expression (4) is identical to the expression for the weight noise power spectrum with the dispersion σ_F^2 going through the RC-filter with a time constant 1/β, that is, the duty cycle of the explosion signal in the frequency band of interest to us (for large β) can be considered distributed with respect to a normal law; then the reverberation distribution will also be normal [3]. In order to check this proposition, experiments were stated, the basic purpose of which was establishment of the correspondence of the degree of adequacy of the probability characteristic obtained on the basis of the adopted model and the characteristic obtained as a result of processing the experimental data.

The experiments were performed under various conditions -- in shallow bays with an average depth of 25-30 meters and in an open lake with a depth of 20 meters (sandy bottom with small slope). There were no waves, wind or current. The set of recordings was made up of 300 realizations lasting 3 to 4 seconds each.

The signal sources were explosive charges and electric detonators, and the receiver was a nondirectional hydrophone, the depth of submersion of which was equal to the depth of the blast of the charge, the configuration of the arrangement of the source in the receiver in the given case can be considered monostatic. The adopted reverberation signals in a wide band were recorded on a tape recorder. Further processing was carried out with the application of the BPF system for various values of the frequency (from 100 to 700 hertz) with a 2-hertz band. Here the realization was broken down into sections 0.5 seconds long.

According to the studies performed in reference [2], the coefficients of the backscattering for the frequency band investigated by us does not depend on the frequency. Therefore the law of decrease in average reverberation level for the narrow-band components with accuracy to a constant factor must correspond to that for the wide frequency band. Hence, it follows that the stationarization of the reverberation process can be accomplished after the Fourier transformation using normalization with respect to the average value of the spectrum in the investigated frequency band. Here it is significant that the magnitude of the spectrum normalized with respect to the average value in the corresponding time interval, in a narrow band at each point in time is considered to be an independent random event, which offers the possibility of proceeding from averaging with respect to time to averaging with respect to frequencies. The statistical processing of the reverberation spectrograms was carried out as follows. In the spectrum of h time interval, an average level was found in the wide band $y_{mean}=(1/n)\sum_{i=1}^{n} y(f_i)$, where n is the number of analysis bands (308); $y(f_i)$ is the value of the spectral density with respect to the frequency f_i. In order to construct the integral distribution law on each spectrogram, the numbers of areas $y(f_i)$ found below the level m y_{mean} were calculated (this representation of the spectral density was the operation of integral stationarization), where $y_{mean}(f_i)$ was assumed to correspond to the given spectrogram, and m assumed 9 values. Then the probability was determined with which the spectral power on any frequency within the limits of the investigated band will be below these levels:

$$F'(m\, y_{cp(1)}) = \frac{n_m}{n},$$

Key: 1. mean

where n_m is the number of spectral bands exceeding the given level. Then averaging was carried out for the probabilities of each level obtained with respect to all of the spectrograms

$$F'(m\, y_{cp}) = \frac{1}{n_1}\sum_{i=1}^{n_1} F_i(m\, y_{cp}),$$

where n_1 is the total number of spectrograms.

According to [3], the energy spectrum of the normally distributed process must be distributed as the square of the normally distributed value, that is,

$$f_n(y) = \frac{1}{\sqrt{2\pi}\sigma_n\sqrt{y}} e^{-\frac{y}{2\sigma_n^2}}, \qquad (6)$$

where $f_n(y)$ is the probability density; $y=x^2$ is the value of the random variable $y(f_i)$expressed in units by its mean, that is, $y = \frac{y(f_i)}{y_{mean}} = m$;

x is the normally distributed variable; σ_n^2 is the dispersion of the normally distributed variable. The integral function of this distribution has the form:

$$F'_n(y) = 2\Phi\left(\frac{\sqrt{y}}{\sigma_n}\right) - 1, \qquad (7)$$

where $\Phi(\sqrt{y}/\sigma_n)$ is the probability integral.

If we substitute the probabilities $f_n(y)$ calculated for various levels m in (7), and we determine the dispersion at each level and average the result with respect to 9 points, we obtain:

$$\sigma_n = \sqrt{0.99\, y_{cp(1)}} \approx \sqrt{y_{cp}},$$

Key: 1. mean

which confirms our studies.

The testing of the correspondence of the distribution (7) to the experimental data with respect to the Kolmogorov criterion gives a probability of 0.997.

Thus, the result we obtained makes it possible to consider that the distribution of the stationary component of the explosion reverberation under shallow water conditions is subject to a normal law.

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ENERGY THEORY OF MULTIBEAM REVERBERATION AND ITS COMPUTER SIMULATION

By V. N. Goncharov, V. V. Ol'shevskiy, pp 43-55

1. The discussion of the problem. The studies of the characteristics of marine reverberation began in the 1940's. In the papers by Yu. M. Sukharevskiy [1-5] and American researchers [6] the energy and partially statistical theory of reverberation was developed for the first time as applied to the dispersing aqueous environment and its boundaries under the assumption that the refraction phenomena in the medium and the effect of reflections from the boundaries can be neglected. However, already in these first papers [5, 6] it was noted that both refraction and reflections from the boundary can have a significant influence on the reverberation characteristics observed under real marine conditions. Further studies published in the 1950's and 1960's were basically devoted to the development of the statistical theory of reverberation: they ended with the publication of the monographs by V. V. Ol'shevskiy [7, 9] and the series of papers by D. Middleton [8]. In the last 10 years the studies of the acoustic properties of the ocean have been performed highly intensely; in our country primarily the studies have been under the direction of Academician L. M. Brekhovskikh [10, 11]; here the primary attention has been given to the differential theoretical and experimental study of individual acoustic phenomena. It is natural that by the end of the 1970's a foundation had been created for the next step -- the construction of the reverberation model with significantly more complete consideration of the acoustic phenomena than was made earlier (see, in particular, survey [12]).

A characteristic feature of marine reverberation is the fact that its properties are influenced both by the systems characteristics and the physical phenomena occurring on propagation of the acoustic waves in an aqueous environment (see Fig 1). This is a significant obstacle on the path of creating a generalized and correct wave model of reverberation which, on the one hand, will make it possible to consider the most significant systems characteristics (the directionalness of the antennas, their movement, the type of emitted signal), and on the other hand, would be well interpretable in physical respects (consideration of the acoustic phenomena, the observed experimental data).

Accordingly, the effort to create in some sense a compromise model of multibeam reverberation in which, on the one hand, the largest number of factors significant in physical respects influencing the formation of the reverberation would be sufficiently simply realizable (for example, on modern computers) and, consequently, it would be constructive. This approach was demonstrated in references [8, 13-19] in which an effort is made to combine in one form or another the beam methods of calculating the characteristics of the acoustic fields in the ocean considering the effect of the systems parameters on the reverberation characteristics. This is natural, for the combination of the energy and the statistical models of reveberation is a significantly more complicated problem, for the solution of which sufficiently developed both energy and statistical models individually are required. Of course, it would be possible immediately to state the problem of creating a generalized mechanical statistical model of multibeam reverberation or determination of the general solution of the wave equation for arbitrary stochastic layered-nonuniform medium with dispersing nonuniformities and boundaries. However, this "global" path, although theoretically enticing, is essentially only decorative, and in our opinion, hardly productive and realizable than the successive investigation based on gradual complication of the reverberation model and the acquisition of experience step by step.

The purpose of this paper is a survey of the existing publications [13-19] pertaining to the studies of the energy characteristics of multibeam reverberation and also the discussion of the basic ideas and methods of solving the problem of determining the energy characteristics of reverberation on a computer.

2. The energy models of multibeam reverberation. Let us formulate first of all the propositions with respect to the acoustic properties of the wave medium and its boundaries which in one form or another are the basis for various models of multibeam reverberation [13-19], and which will be used by us hereafter.

A1. It is considered that the dispersing nonuniformities in the aqueous environment are located in the horizontally oriented infinite layer of finite thickness and (or) on the boundaries of the medium (the water surface and the bottom).

B1. The scattering from the layer and the boundaries is described using the energy scattering coefficients which are considered known from the direct acoustic experiments.

C1. The reflection (in the mirror direction) from the boundaries of the environment is described using the energy reflection coefficients which are considered known from the direct acoustic experiments.

D1. Damping in the medium is considered (in the general case) to depend on the temperature, the salinity, and the hydrostatic pressure, that is, the depth.

E1. It is proposed that the speed of sound in the medium is approximated by a piecewise linear depth function and is constant in the horizontal plane (the case of layered nonuniform deterministic medium).

Let us also formulate the basic propositions with respect to the svstems characteristics which were used when constructing the energy model of multibeam reverberation [17, 18].

A2. Emitting and receiving acoustic antennas are considered matched.

B2. It is proposed that the characteristics of the acoustic antennas are described using the energy angular radiation patterns.

C2. The emitted signal is considered to be narrow band.

Thus, the propositions of A1, B1, C1, D1 and E1 pertain to physical factors, and A2, B2 and C2, to the systems characteristics, for which the energy model of the multibeam reverberation is developed (see Fig 1).

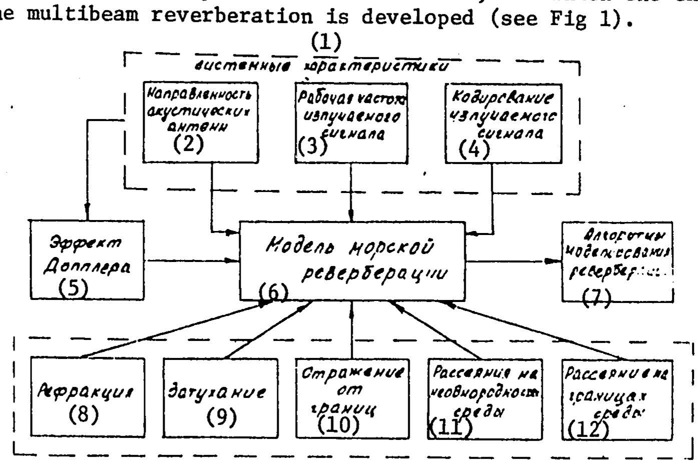

Figure 1

Key:

1. Written characteristics
2. Directionalness of the acoustic
3. Operating frequency of the emitted signal
4. Coding of the emitted signal
5. Doppler effect
6. Model of marine reverberation
7. Algorithms for simulation of reverberation
8. Refraction
9. Damping
10. Reflection from the boundaries
11. Scattering on a nonuniformity of the medium
12. Scattering on the boundaries of the medium

Then we shall discuss the method of calculating the energy characteristics of reverberation in the role of the above-formulated propositions with respect to the acoustic properties of an aqueous environment and the systems parameters.

The characteristics of the acoustic waves are calculated in the beam approximation (geometric optics). Here it is possible to indicate several versions of the calculation of the energy characteristics of reverberation which are based on the indicated approximation (see, for example, [13-16]). First of all, let us note two basic versions of the calculation.

The first version of the calculation -- the beam method -- is based on the breakdown of the entire scattering region into elementary regions (in the case of scattering from the layer, the cross sections in the vertical plane of these elementary regions are rectangles with sufficiently small sides, and in the case of scattering from the boundary, horizontal segments lying on the boundary, the lengths of which correspond to the side dimensions of the indicated rectangles). Then for each elementary region the center is found, the beams are constructed between the emission-reception point and each of the centers and, finally, the acoustic parameters of the beams are determined (the angles of emergence from the radiation point and the angles of arrival at the centers of the cross sections of the focusing factors, propagation times). The energy model of multibeam reverberation in this version is constructed on the basis of summation of the intensities of the signals scattered by the elementary regions and arriving at the reception point in the time intervals equal to the effective duration of the emitted signal.

Thus, the beam method is based on the subdivision of the entire scattering region into elementary regions and subsequent summations of the signals scattered by these elementary regions propagated over various beams, but arriving simultaneously at the reception point.

The second version of the calculation -- the beam method -- is based on breakdown of the entire sector of angles in the vertical plane into radial beams (congruences) at the emission-reception point. Each of these beams combines a set of beams similar to each other with respect to their propagation characteristics (identical number of turns of the beams and reflections of them from the boundaries, the same layers in which these beams are propagated and also the satisfaction of the condition that none of the beams will over the entire trajectory go beyond the bounding beams of the beam, perhaps, with the exception of the caustic regions). For each radial beam, the geometric pattern of propagation of the emitted signal with respect to it is determined, and the cross sections of the signal with the dispersing region are defined (in the case of scattering from a layer the cross sections in the vertical plane are convex polyhedrons, and in the case of scattering from the boundary, horizontal segments lying in the plane of the boundary); in addition, the mutual intersections of the indicated cross sections are determined which correspond to the various beams.

Then for each apex of such a cross section, the acoustic parameters of the beams which pass through these apexes are found (the angles of emergence from the emission point and the angles of arrival at the apexes, the focusing factors, the propagation times). The energy model of the multibeam reverberation in this version is constructed on the basis of summation of the intensities of the signal scattered by the regions corresponding to the cross sections of the emitted signal on propagation of it with the scattering regions and arriving at identical points in time with respect to the various radial beams.

Thus, the method of radial beams is based on the breakdown of the entire sector of angles in the vertical plane into elementary radial beams and subsequent summation of the scattered signals simultaneously arriving at the reception point along these radial beams.

In both of the described methods the directionalness of the radiating and receiving acoustic antennas is taken into account in the form of factors in angular coordinates; the possible angular dependence of the scattering coefficients and the reflection coefficients are also taken into account analogously.

It is possible also to note some other possible versions of calculating the energy characteristics of multibeam reverberation [15], but their basis is (in one form or another) one of the above-described versions (the beam method or the method of radial beams) or a combination of them.

Let us consider the basic relations for determining the energy characteristics of multibeam reverberation from the dispersing layer by the method of radial beams.

Let for the i-th radial beam

$$J_i = \frac{P_A \gamma_H}{4\pi D_i^2} \varphi_H^2(\alpha, \theta_i) f_i \, 10^{-0.1\beta D_i} \quad (1)$$

be the intensity of the direct acoustic field irradiating the scattering region; here P_A is the acoustic power of the transmitting antenna, [symbol missing] is its coefficient of axial concentration, and $\varphi_H(\alpha, \theta_i)$ is the normalized directivity characteristic; f_i is the focusing factor; β is the damping coefficient; D_i is the distance corresponding to acoustic length of path of the i-th beam.

Let us consider the scattering region ΔV_i irradiated by the acoustic signal propagated by the i-th beam in the horizontal plane (this region corresponds to the angle $d\alpha$).

Generally speaking, for multibeam propagation of the sound waves the same region ΔV_i (or part of it) can simultaneously be irradiated by the signals propagated along other beams (with the numbers j, k and so on).

Then obviously the region ΔV_{ij} simultaneously irradiated by the i-th and the j-th beams is defined in the form:

$$\Delta V_{ij} = \Delta V_i \cap \Delta V_j , \tag{2}$$

where

$$\Delta V_{ij} = \Delta V_i , \quad i = j. \tag{3}$$

The power of the signals scattered ΔP_{pij} by the region ΔV_{ij} is:

$$\Delta P_{pij} = K_p \mathcal{J}_i \Delta V_{ij} \varphi_p^2 (\Psi_i , \Psi_j), \tag{4}$$

where K_p is the volumetric scattering coefficient, $\varphi_p^2(\Psi_i, \Psi_j)$ is the indicatrix of scattering of the region ΔV_{ij}, the values of which correspond to the angle of irradiation Ψ_i of the i-th beam and the angle of scattering Ψ_j for the j-th beam.

The intensity of the reverberation $\Delta \mathcal{J}_{pij}$ generated by the scattering in the volume ΔV_{ij} and arriving at the reception point along the j-th beam is defined as follows:

$$\Delta \mathcal{J}_{pij}(\alpha) = \frac{\Delta P_{pij} \gamma_p}{4\pi D_j^2} \varphi_n^2(\alpha, \theta) f_j \, 10^{-0.1\beta D_j}, \tag{5}$$

where γ_p is the concentration coefficient of the dispersing volume ΔV_{ij}, $\varphi_n(\alpha,\theta)$ is the directivity characteristic of the receiving antenna, f_j is the focusing factor, D_j is the distance corresponding to the acoustic length of path of j-th beam. Considering expressions (1), (2), (4) and (5), we obtain:

$$\Delta \mathcal{J}_{pij}(\alpha) = \frac{P_A \gamma_H \gamma_p K_p f_i f_j}{(4\pi)^2 D_i^2 D_j^2} \times \tag{6}$$
$$\times \varphi_H^2(\alpha, \theta_i) \varphi_n^2(\alpha, \theta_j) \varphi_p^2(\Psi_i, \Psi_j) \Delta V_i \cap \Delta V_j \, 10^{-0.1\beta (D_i + D_j)}.$$

It is obvious that the reverberation intensity $\Delta \mathcal{J}_{pij}$ caused by the scattering from the cylindrical region, that is, integration in the horizontal plane, is equal to:

$$\Delta \mathcal{J}_{pij} = \int_0^{2\pi} \Delta \mathcal{J}_{pij}(\alpha) d\alpha. \tag{7}$$

Assuming that

$$\varphi_H(\alpha, \theta_i) = \varphi_H(\alpha) \varphi_H(\theta_i), \quad \varphi_n(\alpha, \theta_j) = \varphi_n(\alpha) \varphi_n(\theta_j), \tag{8}$$

considering (6) and (7), we have:

$$\Delta J_{pij} = \frac{\rho_A \eta_{\text{ип}} \gamma_p K_p f_i f_j}{8\pi D_i^2 D_j^2} \varphi_{\text{и}}^2(\theta_i) \varphi_{\text{п}}^2(\theta_j) \varphi_p^2(\psi_i, \psi_j) V_i \cap V_j \, 10^{-0{,}1\beta(D_i + D_j)} \quad (9)$$

where V_i, V_j are the total scattering volumes, $\eta_{\text{ип}} = \gamma_{\text{и}} / \gamma_{\text{ип}}$,

$$\gamma_{\text{и}} = \frac{4\pi}{\int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \varphi_{\text{и}}^2(\alpha, \theta) \cos\theta \, d\alpha \, d\theta}, \quad (10)$$

$$\gamma_{\text{ип}} = 2\pi / \int_0^{2\pi} \varphi_{\text{и}}^2(\alpha) \varphi_{\text{п}}^2(\alpha) \, d\alpha. \quad (11)$$

For simplicity let us assume that

$$D_i = D_j = D, \quad \varphi_p(\psi_i, \psi_j) = 1, \quad \gamma_p = 1 \quad (12)$$

and let us consider the relation of the scattering volumes V_i, V_j with the cross sections S_i, S_j in the vertical plane

$$V_i = D S_i, \quad V_j = D S_j \quad (13)$$

(here expression (7) is considered).

In the later calculations we shall use the concept of the multibeam reverberation anomaly [13, 14, 16-19] which is defined as follows:

$$\Delta A_{pij} = \frac{\Delta J_{pij}}{J_{po}}, \quad (14)$$

where

$$J_{po} = \frac{\rho_A \eta_{\text{ип}} K_p \Delta H_c c T_0}{16\pi D^3} 10^{-0{,}2\beta D} \quad (15)$$

is the intensity of the single beam reverberation from the layer for a uniform (in the sense of refraction) medium, ΔH_c is the thickness of the dispersing layer, T_0 is the duration of the emitted signal; c is the speed of sound. Then considering (9), (12)-(15) we find:

$$\Delta A_{pij} = \frac{2 f_i f_j}{\Delta H_c c T_0} \varphi_{\text{и}}^2(\theta_i) \varphi_{\text{п}}^2(\theta_j) S_i \cap S_j. \quad (16)$$

Factually ΔA_{pij} is the two-dimensional angular (in the angular coordinates θ_i, θ_j in the vertical plane) spectrum of the reverberation anomaly.

The angular spectrum (16) permits us to determine the characteristics of the multibeam reverberation at the output of the receiving antenna considering the radiation pattern of the transmitting antenna. Here it is necessary to sum the expression (16) with respect to all combinations (i, j) of this radial beam. Then the reverberation anomaly A_p is defined by the expression:

$$A_p = \frac{2}{4 H_c c T_0} \sum_{i,j} f_i f_j \varphi_u^2(\theta_i) \varphi_n^2(\theta_j) S_i \cap S_j. \qquad (17)$$

3. Simulation of the multibeam reverberation. In accordance with the energy model of reverberation developed in item 2, we have created a program for calculating its characteristics [16, 17, 19]. The block diagram for this program, which was called REGOL-76, is presented in Fig 2.

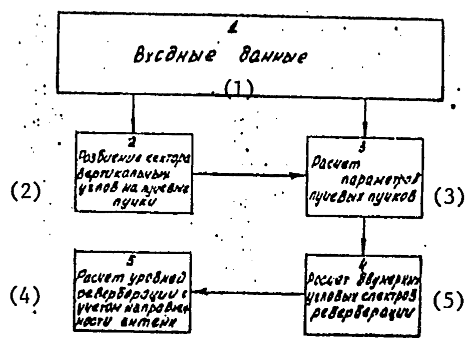

Figure 2

Key:
1. Input data
2. Breakdown of the sector of vertical angles in the radial beam
3. Calculation of the parameters of the radial beams
4. Calculation of the levels of reverberation considering the directionalness of the antenna
5. Calculation of two-dimensional angular reverberation spectra

The input data (block 1) for this program are the following variables: the sound velocity profile, the depth of the transmitting and receiving antenna, the depth at which the upper and lower boundaries of the sound dispersing layer (or layers) are located, the numbers of the zone of the far acoustic illumination (or number of zones), the coefficient of scattering from the layer and the boundary and also the coefficients of reflection from the boundaries, that is, from the surface and the bottom. In addition, for the operation of this program it is necessary to give the signal duration and also to describe the type of radiation pattern of both the receiving and the transmitting antennas. For example, if the radiation patterns are assumed to be rectangular, then it is necessary to describe three variables: the width of the main lobe of the pattern, the direction of the main lobe and also the level of the side lobes. Here the level of the main lobe is assumed to be equal to one (see item 4).

It must be noted that sometimes the survey sector is limited, beginning with the peculiarities of the propagation of sound waves: for example, if we calculated the reverberation level from the far zones of the acoustic illumination, then the "bottom" beams, that is, the beams reflected from the bottom, can be greatly attenuated; therefore they frequently are not taken into account in the calculations. In this case it is necessary to give the magnitude of the survey sector.

The given survey sector is broken down into the radial beams already described above (block 2). The angular width of the radial beams is selected arbitrary, but no more than one degree, for the boundary rays of the beams of this width diverge to significant diseases and it is not always appropriate to neglect the rays included between the boundary rays of such a beam. When selecting the boundary rays of radial beam, it is necessary to be guided by the above-discussed arguments (see item 2). Let us again emphasize that the basic requirement on the boundary rays of the radial beams is the condition of their simultaneous rotation in the same layers (and, consequently, the number of reflections from the interfaces). The algorithm of the second block of the program was constructed under this condition.

Then, in block 3 the parameters of the boundary radial beams are calculated for the selected far zone of acoustic illumination (or for the zones successively). Such values as the horizontal distance to the points of intersection of the beam trajectories with the horizontals into which the travel time to these points and also (for the same points) the values of the derivative required for calculation of the focusing factor (all of the formulas necessary for this are presented in [16]) are broken down with respect to depth of the ocean (in accordance with the breakdown of the sound velocity profile). It is necessary to note that these formulas correspond to the case of piecewise linear approximation of the square of the coefficient of refraction. Using the values obtained, it is possible to calculate the different characteristics of the multibeam reverberation: the level of the reverberation anomaly, its angular spectra, and so on. The calculation of these characteristics is made by formulas (16), (17) (blocks 4, 5).

Let us briefly describe the course of the calculations by these algorithms. A point in time is selected, and the coordinates of the apexes of the cross section of the elementary scattering volume are determined. This is done using the equation which relates the travel time of the beam to some point in space to the refraction coefficient at the point. Then it is checked whether the given elementary volume falls in the scattering layer. In the case that it does, the focusing factors of the apexes of the cross section are calculated (considering the values of the radiation patterns of the antennas and the reflection coefficients from the boundaries). Then the average focusing factor in the given beam is found and, finally, the scattered signals are summed with respect to all the beams incident in the layer. Let us note that on passage of the beams through the caustic zones, the values of the focusing factors have an upper bound of 25 decibels.

4. The conditions and the results of simulating multibeam reverberation. In accordance with the algorithms (2), the structural diagram (3) various energy characteristics of the multibeam reverberation were obtained on the BESM-6 computer using the REGOL-76 program.

The conditions of performing the computer experiments were characterized by the presence of an underwater sound channel and far zones of acoustic illumination. Fig 3 shows the adopted distribution of the speed of sound with respect to depth c(z) and also the arrangement of the acoustic antennas and the scattering layer.

The nondirectional characteristics of the acoustic antennas (in the vertical plane) were assumed to be of the "sector" type (see Fig 4).

Fig 5 shows the graphs of the reverberation anomaly A_{pm} (in decibels) corresponding to the different directional parameters of the acoustic antennas. Here the duration of the emitted signal of rectangular shape was assumed equal to 1 second. The graphs were constructed as a function of the current time which corresponds to the method of calculating the parameters of the radial beam used in the REGOL-76 program.

In Table 1 the results are presented from calculations of the two-dimensional angular spectra of the anomaly of multibeam reverberation. This machine experiment corresponded to observation of reverberation from the second far zone of acoustic illumination for two points in time. When calculating the spectra of the reverberation anomaly, the angular widths of the radial beams at the radiation-reception point were assumed equal to 0.5°. The skipped elements in Table 1 correspond to the values of the reverberation anomalies less than -80 decibels.

Fig 6 shows examples of histograms of the reverberation anomaly calculated for the second far zone of acoustic illumination and corresponding to different directionalness of the acoustic antennas.

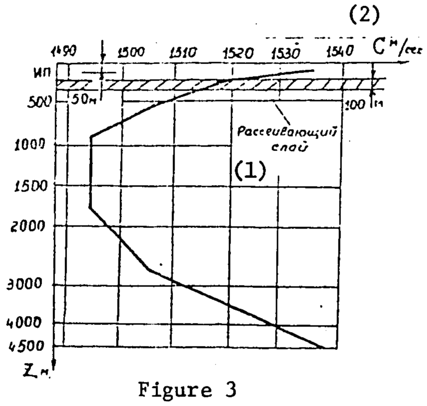

Figure 3

Key: 1. Scattering layer; 2. C, m/sec

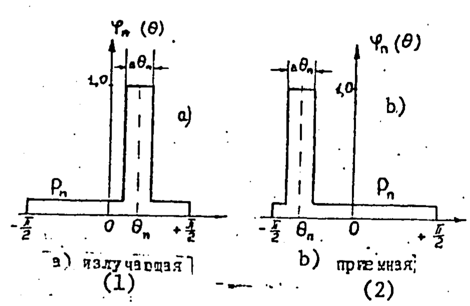

Figure 4

Key: 1. a) emitting; 2. b) receiving

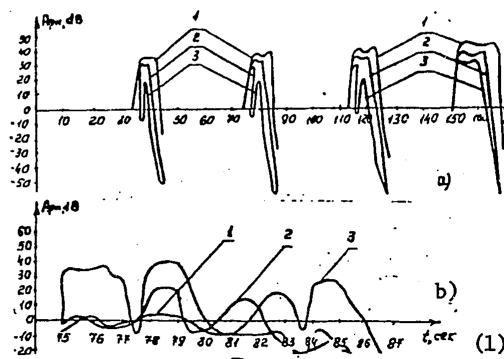

Figure 5

Key:
1. t, sec

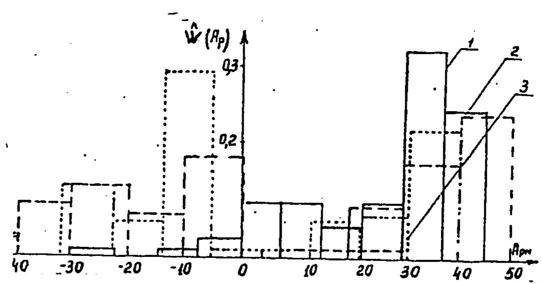

Figure 6

5. Resume. The development of the energy model of the multibeam reverberation presents an urgent problem in the further development of acoustic methods of investigating the ocean. One of the efficient versions of the construction of such a model is the method of radial beams which is implemented in the REGOL-76 program for the BESM-6 computer. By using the REGOL-76 program the following can be determined:

The energy characteristics of the multibeam reverberation with different directionalness of the emitting and receiving antennas;

The two-dimensional angular energy spectra of the multibeam reverberation;

The probability distributions of the levels of multibeam reverberation when the random values of the reckoning times are given.

The authors express their appreciation to Prof Yu. M. Sukharevskiy for discussion of the problem and useful proposals.

Table 1

(1)

[illegible]	-9,5	-8,5	-7,5	-6,5	-5,5	-4,5	-3,5	-2,5	-1,5	-0,5	0,5	1,5	2,5	3,5	4,5	5,5	6,5	7,5	8,5	9,5	
-9,5																					
-8,5																					
-7,5																					
-6,5																					
-5,5																					
-4,5																					
-3,5							1,8														
-2,5								20,2										16,3			
-1,5									20,9												
-0,5										21,3							84,8				
0,5											24,5					80,1	14,2				
1,5												22,1			15,4	21,3					
2,5													0,4		13,5						
3,5																					
4,5												22,1	13,5		20,2						
5,5																22,2					
6,5										21,6							12,1				
7,5								16,3										12,6			
8,5																					
9,5																					

Key:
1. deg

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UDC 551.468.21

SOME PECULIARITIES OF CALCULATING THE ENERGY CHARACTERISTICS OF MARINE MULTIBEAM REVERBERATION FROM BOUNDARIES AND FROM THE SURFACE LAYER

By V. N. Goncharov, pp 55-57

At the present time on the basis of a number of circumstances -- complexity, the necessity for large expenditures, theoretical difficulties in forming the representative sets of realizations, and so on -- computer simulation is being widely used as a tool to obtain the characteristics of marine multibeam reverberation (see, for example [1, 2]).

In reference [2] the basic relations are presented which permit calculation of the energy characteristics of marine multibeam reverberation from the deep water scattering layer, in particular, in the intensity of this type of reverberation. Maintaining the strength of all the proposals with respect to the acoustic characteristics of the aqueous environment and the systems characteristics used in the model and also repeating all of the transformations ((1-8), (10-11)) presented in [2], it is possible to obtain the expression for the intensity of the multibeam reverberation from the ΔS section of the scattering boundary of the aqueous environment (both for the bottom and for the surface). This expression has the following form:

$$\Delta \mathcal{J}_{pij} = \frac{P_A \eta_{\text{ип}} \gamma_p K_{po} f_i f_j}{8\pi D_i^2 D_j^2} \varphi_{\text{и}}(\theta_i) \varphi_{\text{п}}(\theta_j) \times \varphi_p^2(\Psi_i, \Psi_j) \Delta S_{ij} 10^{-0.1\beta(D_i + D_j)},$$

where

$$\Delta S_{ij} = \begin{cases} \Delta S_i \cap \Delta S_j, & i \neq j; \\ \Delta S_i, & i = j, \end{cases}$$

and all the remaining notation is analogous to that used in the expression (9) (see [2]).

As is known, the expression for the intensity of the reverberation from the scattering boundary $\mathcal{J}_{p2po}$ is written as follows:

$$J_{po} = \frac{A_H \eta_{nn} k_{po} H_u c T_o}{16 \pi D^4} 10^{-a_2 \beta D},$$

where k_{po} is the scattering coefficient from the boundary, H_u is the depth of the emitter.

Using the definition of the reverberation anomaly

$$A_{pij} = \frac{\Delta J_{ij}}{J_{po}},$$

and also considering that

$$\Delta S_{ij} = D \Delta L_{ij},$$

where ΔL_{ij} is the vertical cross section of a segment ΔS_{ij} of the scattering surface "illuminated" by the i-th and the j-th radial beams, it is possible to obtain the expression for the anomaly of marine multibeam reverberation from the interface:

$$A_{p2p} = \frac{2}{H_u c T_o} \sum_{i,j}^{n} f_i f_j \varphi_n^2(\theta_i) \varphi_n^2(\theta_j) \Delta L_{ij},$$

(here the expressions (12) were also taken into account, see [2]).

Now let us proceed with the investigation of the expression for the anomaly of marine multibeam reverberation from the surface scattering layer. Its form, with one exception, is entirely analogous to the form of the expression (9) see [2]. As follows from Fig 1, when studying reverberation from the surface layer, it is necessary to consider the "autointersections," that is, the intersections of parts of the beam with each other. Consequently, the term ΔS_{ij} in formula (9) is altered to a sum of the following type: $\sum_{k,\ell=1}^{2} \Delta S_{ijk\ell}$, where

$$\sum_{k,\ell=1}^{2} \Delta S_{ijk\ell} = \begin{cases} \left.\begin{array}{ll} \Delta S_{ik}; & k=\ell \\ \Delta S_{ik} \cap \Delta S_{i\ell}; & k \neq \ell \end{array}\right\} & i=j \\ \left.\begin{array}{ll} \Delta S_{ik} \cap \Delta S_{jk}; & k=\ell \\ \Delta S_{ik} \cap \Delta S_{i\ell}; & k \neq \ell \end{array}\right\} & i \neq j \end{cases}$$

Using expressions (5) and (6) and the REGOL-76 program, the time relations were calculated for the anomalies of three types of reverberation: surface, the scattering layer next to the surface and also the deep water scattering layer for the second far zone of the acoustic illumination (the distribution of the speed of sound with respect to depth and location of the acoustic antennas is analogous to that presented in Fig 3, see [2]).

The depth of the surface layer is 40 meters; both the emitting and the receiving antennas are nondirectional and matched. These relations are presented in Fig 2.

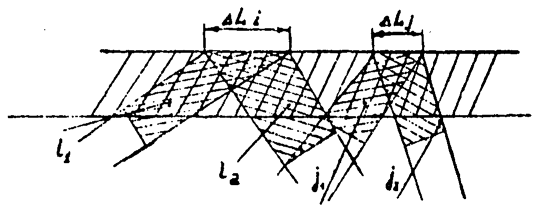

Figure 1. Intersections of the radial beams in the surface layer

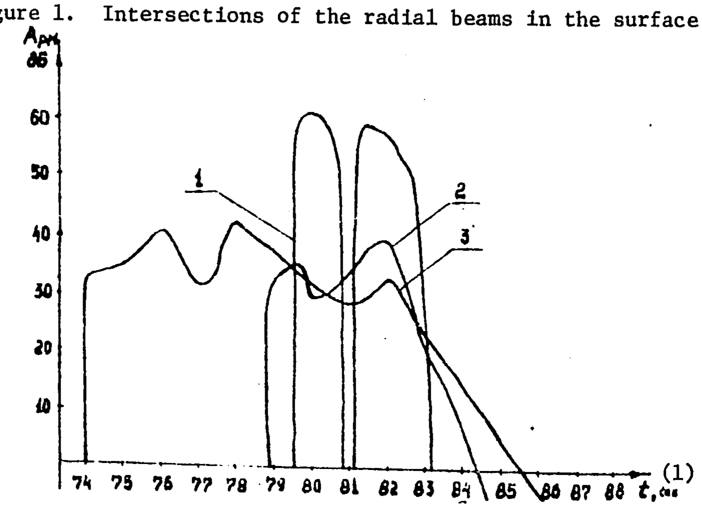

Figure 2. Reverberation anomaly A_{pm}: 1 -- from the surface; 2 -- from the surface scattering layer; 3 -- from the deep water scattering layer

Key:

1. sec

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SPECTRAL AND CORRELATION PROPERTIES OF MARINE REVERBERATION WITH SCANNING EMISSION

By F. I. Pedenko, pp 58-60

In order to consider the effect of scanning on the probability characteristics of reverberation as the initial expressions describing the autocorrelation function of reverberation on displacement of acoustic antennas, according to [1] we assume:

$$B_n(\tau)=\langle n_1\rangle\int_0^{2\pi}\langle a^2(\alpha)\rangle W(\alpha)\int_{-\infty}^{\infty}S(t,\alpha)S(t+\tau,\alpha)\,dt\,d\alpha. \quad (1)$$

Here: $\langle n_1\rangle$ is the intensity of the reverberation; $W(\alpha)$ is the distribution density of the elementary signals $S(T, \alpha)$, the parameter α of which characterizes the overall time shape of the signals received from the α direction; $\langle a^2(\alpha)$ is the coefficient which takes into account the mutual effect of the emitting $\varphi_u(\alpha-\alpha_u)$ and receiving $\phi_\Pi(\alpha-\alpha_\Pi)$ directional characteristics oriented in the α_u and α_n directions respectively:

$$\langle a^2(\alpha)\rangle=\langle a^2\rangle\varphi_u^2(\alpha-\alpha_u)\varphi_n^2(\alpha-\alpha_n). \quad (2)$$

In connection with the fact that during scanning the scanning sector is wider than the main lobe of the receiving directional characteristic $\phi_\Pi(\alpha-\alpha_\Pi)$, it is possible to set $\varphi_u^2(\alpha)=1$. In addition, it is necessary to introduce the factor $\gamma(\alpha)$ into expression (2), which takes into account the variation of the damping of the reverberation signals for different α as a result of variation in time of the interference pattern of the irradiation of the scatterers. As a result, we obtain:

$$\langle a^2(\alpha)\rangle=\langle a^2\rangle\gamma(\alpha)\varphi_n^2(\alpha-\alpha_n). \quad (3)$$

Substituting (3) in (1) and proposing a narrow receiving directional characteristic, that is, close to the δ-function, we obtain:

$$B_n(\tau,\alpha_n)=\langle n_1\rangle\langle a^2\rangle\gamma(\alpha_n)W(\alpha_n)\int_{-\infty}^{\infty}S(t,\alpha_n)S(t+\tau,\alpha_n)\,dt. \quad (4)$$

The reverberation spectrum according to [1], will be:

$$G_p(\omega,\alpha_n)=2\langle n_1\rangle\langle a^2\rangle\gamma(\alpha_n)W(\alpha_n)\left|\int_{-\infty}^{\infty}S(t,\alpha_n)e^{-j\omega t}dt\right|^2. \quad (5)$$

Beginning with an analysis of the operation of the linear grating during scanning, the sounding sending in the direction α_{Π} in the far zone can be represented by the following expression:

$$S(t,\alpha_n) = \sum_i \Pi(t+\Delta\tau_{i\alpha_n})\cos\omega_0[t+\Delta\tau_{i\alpha_n}+\tau_i(t+\Delta\tau_{i\alpha_n})], \quad (6)$$

where

$$\Pi(t) = \begin{cases} 1 & \text{for } t\in[-\frac{T}{2},\frac{T}{2}]; \\ 0 & \text{for other } t, \end{cases} \quad (7)$$

T is the duration of the scanning cycle (the displacement of the compensation angle β from $-\pi/2$ to $\pi/2$ from the normal to the lattice; $\tau_i(t)$ is the variable delay:

$$\tau_i(t) = i\frac{\pi}{\omega_0}\sin\beta, \quad (8)$$

i is the number of the emitter; $\Delta\tau_{i\alpha_{\Pi}}$ is the spatial delay:

$$\Delta\tau_{i\alpha_n} = i\frac{\pi}{\omega_0}\sin\alpha_n \quad (9)$$

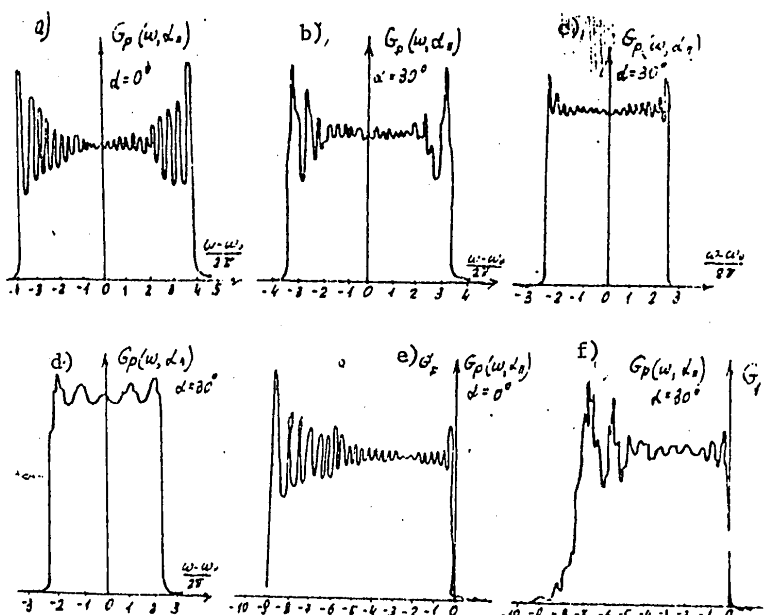

Figure 1

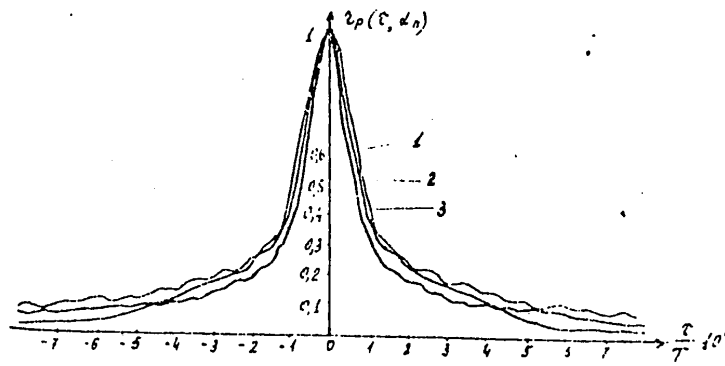

Figure 2

For the calculations of the spectra, two laws of variation of the concentration direction β were considered: -- uniform scanning (uniform-circular displacement of the angle β in space):

$$\beta_{\text{равн}}(t) = \Omega t = \pi \frac{t}{T}, \qquad (10)$$

arc sin -- scanning (linear dependence of the variation of the variable delays $\tau_i(t)$ on time t):

$$\beta_{\arcsin}(t) = \arcsin \frac{2t}{T}. \qquad (11)$$

For the two types of scanning, the spectra (Fig 1) and the correlation coefficients of the reverberation envelope (Fig 2) were calculated:

$$r_n(\tau,\alpha_n) = 1/\left(\int_{-\infty}^{\infty} A^2(t,\alpha_n)\,dt\right)\int_{-\infty}^{\infty} A(t,\alpha_n)A(t+\tau,\alpha_n)\,dt, \qquad (12)$$

where $A(t, \alpha_\Pi$ is the envelope of the signal $S(t, \alpha_\Pi)$. For the calculations T=10 sec, and n=28 were taken. The calculations were performed for symmetric (i=-28.28) and asymmetric (i=0.56) variation of the time delays with respect to the center of the antenna. The presented calculations indicate that within the limits of a sufficiently large scanning sector, to ±45°, the reverberation spectra and the correlation functions of the envelopes vary insignificantly, and for arc sin scanning they remain constant. With asymmetric variation of time delays with respect to the center of the antenna, a shift of the central spectral frequency takes place, which must be taken into account when designing the processing systems.

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UDC 621.391.16

SYNTHESIS OF THE INDETERMINACY FUNCTION WITH RESPECT TO THE CROSS SECTION IN THE PLANE OF THE DOPPLER SHIFTS

By K. B. Krukovskiy-Sinevich, V. P. Cherednichenko, pp 60-61

Under specific conditions of isolation of the echo from moving targets, we are interested only in the properties of one cross section of the indeterminacy function of the sounding signal, namely the cross section in the r=0 plane. If for description of the effect of the movements it is sufficient to use the interdeminacy function of Woodworth, then the problem reduces to finding the complex envelopes satisfying the integral equation

$$\int_{-\infty}^{\infty} U^2(t)e^{j\Omega t}dt = R_0(\Omega), \qquad (1)$$

where U(t) is the modulus of the complex envelope, $R_0(\Omega)$ is the given shape of the cross section of the indeterminacy function.

It must be noted that the problem investigated here does not have a unique solution, for from (1) it is possible to determine only the square of the modulus of the complex envelop of the signal.

Finding such a signal for which the mean square error in approximating the realized function $R(\Omega)$ and the given $R_0(\Omega)$ is minimal, for satisfaction of the condition $U^2(t)\geq)$ turns out to be fruitful. Let us propose that the signal energy is constant. In this statement the definition of $U^2(t)$ reduces to the connected one-way variation problem of minimizing the functional [1]

$$\Delta^2 = \int_{-\infty}^{\infty} |R_0(\Omega) - R(\Omega)|^2 d\Omega = \int_{-\infty}^{\infty} |R_0(\Omega) - \int_{-\infty}^{\infty} U^2(t) e^{j\Omega t} dt|^2 d\Omega. \qquad (2)$$

Using the Gibbs limit [2] to solve the investigated variation problem, we obtain:

$$U^2(t) = Re[U_0(t)] - \lambda, \qquad (3)$$

for

$$Re[U_0(t)] - \lambda > 0;$$

and $$U^2(t) = 0,$$

for $$Re[U_0(t)] - \lambda < 0,$$

where the coefficient λ is determined from the condition $\int_{-\infty}^{\infty} U^2(t)\,dt = const.$

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UDC 621.396.33

USE OF THE PROPERTIES OF INSTANTANEOUS FREQUENCY FOR MEASURING THE STATISTICAL CHARACTERISTICS OF THE HYDROACOUSTIC SIGNALS

By S. A. Bachilo, V. B. Vasil'yev, G. M. Makhonin, pp 61-64

A study is made below of the problem of measuring the doppler shift of the frequency $\Delta\omega_d$ (with respect to the frequency ω_0) of the pulse narrow-band signals against a background of random noise. It is assumed here that the signal lasting t_c appears at an arbitrary point in time in the measurement interval $0-T(T_c \ll T)$, the law of variation of its frequency $\Delta\omega_c(t)$ is a sufficiently smooth function, so that the n-th derivative $\Delta\omega_c^{(n)}(t)$ exists where $\sup/\Delta\omega_c^{(n)}(t)/\leq\varepsilon$, where ε is a small value; the noise is a nonstationary normal random process.

The structural diagram of the doppler frequency shift meter based on using the characteristics of the instantaneous frequency and its derivatives is presented in Fig 1.

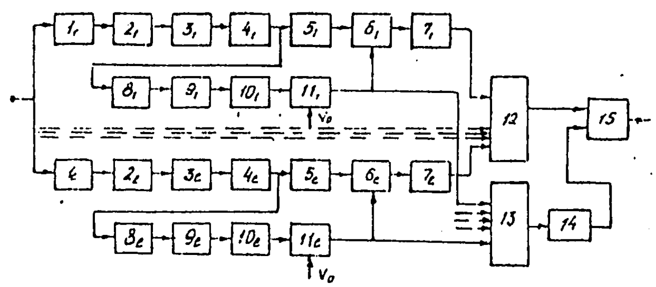

Figure 1. Doppler frequency shift meter:
1_1 to 1_e -- band filter; 2_1 to 2_e -- limiters; 3_1 to 3_e -- frequency detectors; 4_1 to 4_e, 5_1 to 5_e, 10_1 to 10_e -- low-frequency filters; 6_1 to 6_e -- switches; 7_1 to 7_e, 14 -- integrators; 8_1 to 8_e -- n-th order differentiation units; 9_1 to 9_e -- double-threshold amplitude discriminators; 11_1 to 11_e -- threshold stages; 12, 15 -- adders; 13 -- module for selecting the channel number.

The device depicted in Fig 1 contains ℓ measuring channels of identical structure (the modules 1_j to 11_j, j is the channel number) distinguished only by the characteristics of the band filters 1_1 to 1_e. These filters have central frequencies ω_1 to ω_e and cover the entire range of possible values of the doppler frequency shift of the signals. The operation of the device directed in Fig 1 is conveniently considered in the example of one of its channels (the j-th channel). The input voltage through the band filter I_j reaches the limiter 2_j realizing rigid two-way limitation of it. The limiting voltage is fed to the input of the frequency detector 3_j, at the output of which a voltage is formed proportional to the deviation of the instantaneous frequency of the output voltage of the band filter I_j with respect to the central frequency ω_j. This voltage goes through the low-frequency filters 4_j and 5_j to the signal input of the switch 6_j and simultaneously through the filter 4_j to the input of the differentiation module 8_j realizing n-th order differentiation. The input voltage of the module 8_j goes to the double threshold amplitude discriminator 9j which generates the rectangular constant amplitude pulses in the same time intervals where the voltage at its input is inside the interval Δu_j between two threshold levels. On satisfaction of the inequality $\sigma_{8jcn}<\Delta U_j<\sigma_{8jn}$, where σ_{8jcn}, σ_{8jn} are the mean square values of the fluctuations of the output voltage of the differentiation module 8_j in the presence and absence of a signal, correspondingly, the process at the input of the amplitude discriminator 9_j with high probability will be inside the interval ΔU_j in the presence of a signal and outside this interval in the absence of a signal. The pulses from the output of the discriminator 9_j are smoothed by the low-frequency filter 10_j and go to the threshold stage 11_j which generates pulses but open the switch 6_j when the output voltage of the filter 10_j exceeds the threshold level V_0. The output voltage of the switch 6_j is averaged by the integrator 7_j. The value of the process at the output of the module 7_j at the time $T+T_c$ of the end of the measurement interval is the estimate of the deviation of the signal frequency from the central frequency ω_j of the filter I_j. The pulses from the output of the threshold stages 11_1 to 11_e go to the input of the module for selecting the number of the channel 13. On appearance of a pulse at the j-th input of the module 13, a voltage is formed at its output proportional to $\Delta\omega_j=\omega_j-\omega_0$. The output voltage of the module 13 is averaged by the integrator 14 and it is summed in the module 15 with the output voltage of the adder 12. The value of the process at the output of the adder 15 at the time $T+T_c$ of the end of the measurement interval is the estimate $\Delta\omega_d^*$ of the desired doppler frequency shift. Assuming that the processes in the different channels of the device in Fig 1 are statistically independent, it is easy to demonstrate that the variation coefficient β of the estimate $\Delta\omega_d^*$ of the doppler frequency shift of the signal is defined by the expression:

$$\beta=\sqrt{\frac{2\pi}{T_c}}\cdot\frac{\sqrt{\frac{\rho_1}{\ell}\{(1-\rho_1)(\frac{\ell^2}{12}[8-\frac{\ell^2}{\ell}]\Omega^2+(\ldots)^2\}+\rho_1^2(1-\beta)(\ldots)[\text{illegible}]}}{\rho_2[\frac{\Omega}{\ell}]+(\Delta\omega_j-\Delta\omega_\ell)(1-e^{[\text{illegible}]})]},\quad(1)$$

where $\Omega \cdot 2\Delta\omega_{d\ max}$ is the range of possible doppler shifts of the signal frequency; j is the number of the channel containing the signal; $\theta=(T+T_c)/T_c$ is the off-duty factor; $Q=\sqrt{2E/N_0}$ is the signal/noise ratio at the input of the device in Fig 1, E is the signal energy, N_0 is the spectral density of the noise power; $K_0(Q)$ is the coefficient which depends on the signal/noise ratio; τ is the correlation time of the process at the output of the low-frequency filter 5_j; P_1, P_2 are the probabilities of finding the switch 6_j open in the absence and presence of a signal in the j-th channel, respectively; $\mu=\Omega T_c/\pi\ell$.

Fig 2 shows the graphs of the variation coefficients $\beta(Q)$ of the estimates $\Delta\omega_d^*$ as a function of the signal/noise ratio Q obtained on indirect averaging of the instantaneous frequency (graph 1) and when using the device depicted in Fig 1 (graph 2 -- $\ell=1$, graph 3 -- $\ell=3$).

The functions $\beta(Q)$ were calculated for $T_c=25$ msec; $\Omega/2\pi=1500$ hertz; $\theta=10$; $\Delta\Omega_4/\Omega=0.07$; $\Delta\Omega_d/\Omega=0.07$; $\Delta\Omega_\sigma/\Omega=0.05$; $\Delta\Omega_{10}/\Omega=0.03$ and $\Gamma=0.2$.

As is obvious from Fig 2 the measurements of $\Delta\omega_d$ realized by direct averaging of the instantaneous frequency in the interval of T to T_c turns out to be in practice unreliable ($\beta\gtrsim1.9$) for any signal/noise ratios. When using the device in Fig 1 the random error in determining $\Delta\omega_d$ can be significantly diminished. Thus, when $\ell=1$ the device in Fig 1 insures $\beta\lesssim0.15$ for $Q\gtrsim17$.

In cases where it is not possible to insure a signal/noise ratio of this sort, and $\Delta\omega_d$ must be measured with the same accuracy, it is necessary to use a device of the type of Fig 1 with a large of channels. For example, for $\ell=3$ the variation coefficient will become less than 0.15 at $Q\gtrsim13$.

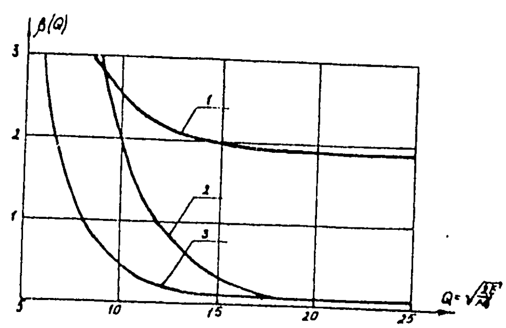

Figure 2. Random measurement error

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POSSIBILITY OF USING THE DIFFERENTIAL ENTROPY WHEN REPRODUCING AN UNKNOWN SIGNAL DISTRIBUTION DENSITY

By A. A. Spivak, pp 64-66

In a number of problems the processing of the acoustic information, in particular, the recognition of signals, various procedures for reproducing the signal probability density by using a polygaussian model have become widespread:

$$p(\vec{x}/H_j) \simeq \sum_{i=1}^{N} \alpha_i W(\vec{x}, \vec{m}_i, \vec{K}_i), \qquad (1)$$

where $W(\cdot)$ is the gaussian density with a vector of the mathematical expectation $\vec{m}_i$ and a covariance matrix $\vec{K}_i$; α_i are the weight coefficients ($\sum_{i=1}^{N} \alpha_i = 1$; $\alpha_i \geqslant 0$; $i=\overline{1,N}$); H_j is the notation for the j-th signal.

The estimation of the value of N -- the number of components in the model (1) which during recognition of the signals is determined from the condition of minimum recognition error γ -- is of special interest.

In [1] one of the methods of formalizing the choice of the optimal value of the number of components in (1) N_{opt} is proposed. For this purpose, a successive estimation is made of the parameters $A^{(t)}$ for N=1,2,3,... For each value of N, the differential entropy is calculated

$$H(N) = -\int_D p(\vec{x}/H_j) \log p(\vec{x}/H_j) d\vec{x}, \qquad (2)$$

which for the distribution (1) has the form:

$$H(N) = \frac{1}{2} \sum_{i=1}^{n} \alpha_i [M \ln(2\pi i) + \sum_{\ell=1}^{M} \ln \sigma_{i\ell}^2] + \Delta H,$$

where $\sigma_{i\ell}^2$ is the partial dispersion of the i-th term of the model (1) in the case where $\vec{K}_i$ is the diagonal covariation matrix; $\Delta H=0$ to $-\sum_{i=1}^{N} \alpha_i \ln \alpha_i$. The presence of two trends -- a decrease in (2) as a result of efficient breakdown of the sample and an increase in (2) as a result of a decrease in the volumes of the subsamples and, consequently, an increase in the corresponding factors for the approximate dispersions -- cause the existence of N_{opt}. Let us estimate the possibility of using (2) as the criterion of the procedure for reproducing the density, skipping the direct calculation of the recognition errors. For this purpose after obtaining the series H(1), H(2), $H(N_{max})$ for the known distribution probability density of the signal $p^*(\vec{x}/H_j)$ we shall calculate the normalized mean square error

$$\delta = \int_{-\infty}^{\infty} \cdots \int \frac{[p(\vec{x}/H_j) - p^*(\vec{x}/H_j)]^2}{[p^*(\vec{x}/H_j)]^2} d\vec{x}. \qquad (3)$$

The results of calculating the values of H, δ and γ for a class of acoustic signals, the true probability density of which in accordance with (1) has $N_{opt}=3$ are presented in Fig 1. When calculating γ, the approximating model (1) with completely known parameters for the second class of signals is adopted. The value of γ (for it and the value of H in Fig 1 the cross-

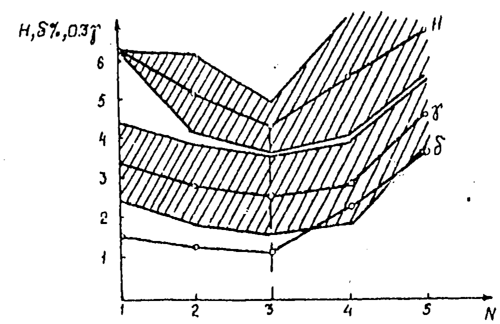

Figure 1. Experimental relations illustrating the accuracy of reproducing the acoustic signal distribution

hatched region shows the confidence intervals at the 0.95 level) was calculated by the method of statistical tests. In the given example all three characteristics have a minimum at the point $N_{opt}=3$. Analogous relations obtained for other classes of signals confirm and substantiate the possibility of using expression (2) for determination of the number of components in the model (1) on reproduction of the distribution probability density of the acoustic signals.

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PROCESSING THE RESULTS OF A REGRESSION EXPERIMENT WITH CORRELATED ERRORS IN THE INPUT AND OUTPUT SIGNALS

By I. G. Dobrovol'skiy, V. V. Fedorov, pp 66-69

In this paper a number of algorithms are proposed for processing information with a different degree of informativeness about the input and output noise and signals in the active and passive experimental regime.

1. Active Experimental Regime. The experimenter gives input variables x_{0i} (the experimental regime) with some error. This corresponds to the following regression problem:

$$\left.\begin{aligned} y_i &= \eta(x_i, \theta) + \nu_i, \quad i = \overline{1,n} \\ x_i &= x_{0i} + h_i \end{aligned}\right\}, \tag{1}$$

where h_i are the errors in the assignment of the input variables; $\eta(x_i, \theta)$ is the known function; $\theta^T = \theta_1, \ldots, \theta_m$ is the vector of the unknown parameters; ν_i is the error in observing the output variable; $x = x_1, \ldots, x_k$ is the number of controllable variables.

Let us generalize the results of [1] in the case of $E[h_i \nu_i] = k^2 d'(x_{0i})$, E is the mathematical expectation symbol, $d'(x_{0i})$ is the covariation matrix. Let us consider that for p, q, r=1,...,k

$$E[h_{ip} h_{iq} h_{ir}] = \xi^3 C_{ipqr}; \quad E[h_i^2 \nu_i] = k^3 d''(x_{0i}), \tag{2}$$

where C is limited, ξ, k are constants. The experimenter has values of y_i and x_{0i} at his disposal ($i = \overline{1,n}$ is the number of points at which the measurements were performed). Therefore in order to construct the regression experiment it is necessary to determine some provisional distribution times $P(y)/x_{0i}$). Forthe i-th observation, we have:

$$y_i = \eta(x_{0i} + h_i, \theta) + \nu_i. \tag{3}$$

Hereafter the following expression will be of interest to us:

$$E[y] = \eta(x, \theta), \quad Var[y] = \delta(x, \theta) \tag{4}$$

(Var is the symbol of taking the second moment.)

It is easy to show that

$$E[y_i] = \int \eta(x,\theta)\rho(x - x_{oi}/x_{oi})dx \qquad (5)$$

and

$$Var[y_i] = \iint (y - E[y_i])\rho(v/x,\theta)\rho(h/x_{oi})dxdy. \qquad (6)$$

Expanding (5) and (6) in a Taylor series in the vicinity of the point x_{0i}, it is possible to obtain calculated expressions for finding the estimates of the parameters θ.

The iteration procedure is defined as follows:

$$U_s(\theta_{s+1}) = \min U_s(\theta), \qquad (7)$$

where

$$U_s(\theta) = \sum_{i=1}^{h}[y_i - \Psi(x_{oi},\theta)]^2\lambda(x_{oi},\theta_s).$$

Let us consider the statistical properties of the estimate and the convergence of the procedure. Let us propose that $\eta(x,\theta)=f^T(x)\theta$. Then the solution of (7) is equivalent to the solution of the system mxn $dv_s(\theta)/d\theta$, giving

$$\theta_{s+1} = M^{-1}(\theta_s)Y(\theta_s), \qquad (8)$$

where

$$M(\theta_s) = M^{-1}(\theta_s)Y(\theta_s), \quad Y(\theta_s) = \sum_{i=1}^{h}\lambda(x_{oi}\theta_s)\Psi(x_{oi})\Psi^T(x_{oi}),$$

$$\Psi_\alpha(x_{oi}) = f_\alpha(x_{oi}) + \frac{1}{2}\xi^2 tr\{d(x_{oi})[\frac{\partial^2 f_\alpha(x)}{\partial x \partial x^T}]_{x=x_{oi}}\}.$$

If (7) converges, then lim $\theta_s=\theta^*$, and if $\lambda(x_{0i},\theta)$ is continuous with respect to θ in the vicinity of θ^*, then θ^* coincides with the solution θ_n of the equation

$$\theta = M_n^{-1}(\theta)Y_n(\theta). \qquad (9)$$

It is possible to state that under certain assumptions there is a series of solutions $\{\theta_n\}$ in (9) which almost certain converge to θ_{true}. Then if (9) has a unique solution $\hat{\theta}_n$ for each $n \gg n_0$, the sequence $\{\hat{\theta}_n\}$ almost certainly converges to θ_{true}.

Let us consider the convergence of the iteration procedure. Let (9) have the solution $\hat{\theta}\in\Omega$; then if the initial approximation is selected in a sufficiently small vicinity of $\hat{\theta}_n$, for convergence of the iteration procedure it is sufficient to check that

$$\max_\alpha \lambda_\alpha[L_n^T L_n] < 1,$$

where $\lambda_\alpha(A)$ is the α-th eigenvalue of the matrix A

$$L_n = \left[\frac{\partial M_n^{-1}(\theta) Y_n(\theta)}{\partial \theta}\right]_{\theta=\theta_n}.$$

II. Passive Experimental Regime. The input variables are given from the outside (the experimental regime). The experimenter observes the values of x_{0i} with an error h_i. Let us consider the experiments of the following type:

$$\left.\begin{array}{l} y_i = \eta(x_{0i}, \theta) + \nu_i, \quad i = \overline{1, n} \\ x_i = x_{0i} + h_i \end{array}\right\}. \qquad (10)$$

The experimenter knows the pairs y_i and x_i; it is necessary first of all to estimate the parameters θ. The given regression problem has a very long history. The set of different methods of finding the estimates was proposed (see, for example [28]). The majority of these methods are unsatisfactory and lead to inconsistent estimates. As was demonstrated in [4], the causes of this lie in the fact that without calling on any additional information in the regression problem (12) the consistent estimates cannot in general be constructed. Here a study will be made of two approaches leading to consistent estimates. For simplicity the errors h_i and ν_i will be assumed to be independent. The generalization to the case of dependent errors can be made, in accordance with the preceding section.

Information on Multiresponse Regression. In many experiments the hypothesis is highly plausible that the values of x_{0i} increase monotonically (or they are monotonically ordered in the multidimensional case). Here it is natural to use the following approximation:

$$x_{0i} = \Psi(\gamma, t_i), \qquad (11)$$

where γ are the unknown parameters; t_i is a fictitious variable, $\Psi(\gamma, t)$ is a function that increases monotonically with respect to t. As soon as the parametric representation has been introduced for x_{0i}, the problem is transformed to the problem

$$\left.\begin{array}{l} y_i = \eta[\Psi(\gamma, t_i), \theta] + \nu_i \\ x_i = \Psi(\gamma, t_i) + h_i \end{array}\right\}. \qquad (12)$$

The problem (12) has been well investigated (see, for example, [5]) and does not prevent special difficulties either in the theoretical-statistical aspect or in the computational aspect.

Evaluation of an Infinitely Large Number of Random Interfering Parameters. The values of x_{0i} can be interpreted as random interfering parameters with some (generally speaking, unknown) distribution function $F(x_0)$. The general theoretical studies of this class of problems from the point of view of the method of maximum plausibility were performed in [4]. Here, some of these studies will be applied to our regression problem (12).

Let the densities P(h), P(ν) be known (for simplicity the same for all x_{0i}), and let x_0 be distributed by the law $P(x_0)$. Then the pair y_i, x_i has the following distribution density (the existence of the latter is assumed):

$$p(y,x) = \int \delta(x-x_0-h)\delta[y-\eta(x,\theta)-\nu]p(h)p(\nu)dh\,d\nu\,dF(x_0) = \int p(y-\eta(x,\theta))p(x-x_0)dF(x_0),$$

where δ is the delta function.

The estimates of the maximum plausibility are defined as follows

$$(\hat{\theta}_n \hat{F}_n) = \mathrm{Arg\,inf} \prod_{i=1}^{n} p(y_i, x_i), \quad \theta, F \in \Phi,$$

where ϕ is the class of admissible distributions $P(x_0)$. In [4] it is demonstrated that the estimates area consistent. In cases where the number of observations is small, satisfactory estimates can be obtained only for parametric description of the set Φ.

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IMPROVEMENT OF THE INFORMATION PROCESSING RATE IN HYDROACOUSTIC SYSTEMS

By V. V. Gritsyk, E. R. Zlatogurskiy, V. N. Mikhaylovskiy, pp 69-71

In this paper a study is made of the highly effective algorithms for processing and recognizing patterns with parallel structure. Estimates of the effectiveness of these algorithms are presented.

The increase in rate and quality of the information processing consists in breaking down the information processing into interdependent groups of operations and the execution of these groups parallel in time [1]. Then the algorithm can be represented on each i-th level of parallel subdivisions in the form of a finite number of mutually independent substates of operations $A_1^i, A_2^i, \ldots, A_{pi}^i$ where $\bigcap_s A_s^i = \emptyset, A^i \subseteq A, A^i = A_1^i \cup A_2 \cup \ldots \cup A_{pi}^i$, where A is the set of all operations. The system for selecting, transmitting, storing, processing and recognizing patterns is illustrated in Fig 1, where: ИС is the distorting system (the hydraulic medium); $\theta(x,y)$ is additive noise; KI is the receiver and coder in the information transmission system in space with respect to the communications channel (channel$_s$); $R(x,y)$ is the noise in the channel$_s$; K2 is the coder in the information transmission system in time, that is, storage on a carrier (channel$_t$); DK is the decoder of the information storage system; KИ is the pattern correction device which decreases the effect of the ИС, $\theta(x,y)$, and $R(x,y)$; ГП is the geometric conversion unit for compensation for the distortions of the geometric shapes and sizes introduced by the ИС, and for normalization of the patterns; P is the pattern recognition device; $S(x,y)$ is the interference determined by the defects in the thermal plastic carrier.

In Fig 2, a, we have the binary pattern of a rectilinear emitter [2]; in Fig 2, b the pure information carrier is shown with defects inherent in it denoted by two gradation levels with respect to brightness; the pattern of the rectilinear emitter is presented in Fig 2, c. After processing (ДК-КИ-ГП) the pattern assumes the form (Fig 2, d) convenient for recognition. In all the steps of selection, transmission, storage and processing of the information and also pattern recognition, algorithms and specialized parallel computers are used, which permits the expenditures of machine time to be reduced by several orders by comparison with the all-purpose computer. The recording and reading of the information from the thermoplastic are realized using the redundant coding methods. Pattern recognition is based on the pattern spectrum method [1].

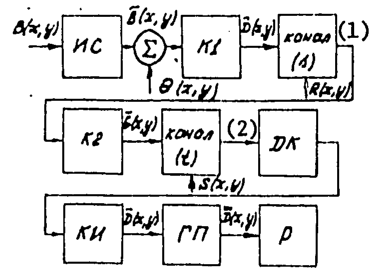

Figure 1. Hydroacoustic pattern processing system
[Notation described in the text]

Key:
1. Channel(s)
2. Channel(t)

The operation of these computing devices is organized so that the same instructions will be executed in the majority of the processes simultaneously with common control.

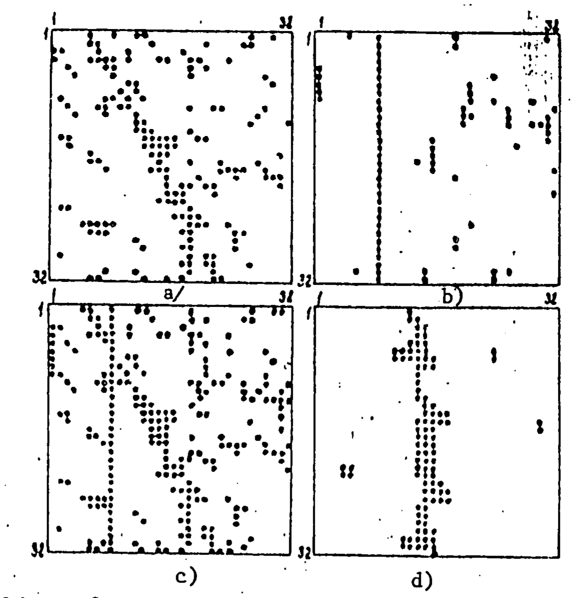

Figure 2. Rectilinear emitter pattern.

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UDC 621.391

SELECTION OF THE INTEGRATION TIME WHEN MEASURING CORRELATED MULTIBEAM SIGNAL FUNCTIONS

By V. P. Akulicheva, S. V. Skripchenko, pp 71-74

In the correlation analysis of multibeam noise signals received against a background of interference, as the integration time increases, two opposite trends from the point of view of isolating the useful signal at the correlator output are observed: a decrease in the dispersion at the channel output as a result of averaging of the signal and noise fluctuations and a decrease in the level of the useful components as a result of averaging it with respect to the fluctuations of the channel parameters and as a result of the doppler effect. It is possible to show that the basic factor decreasing the signal correlation at the channel output is fluctuation of their phases. In this case it is expedient to use the correlator in which the required signal/noise ratio of the output will be insured as a result of averaging of the envelope of the correlation function calculated for a comparatively small integration time (Fig 1). Let us consider the problem of optimization of the integration time in such a correlator beginning with the condition of insurance of maximum probability of proper detection (D) of the maximum of the correlation function of the input signals at the point 3 of the diagram in Fig 1.

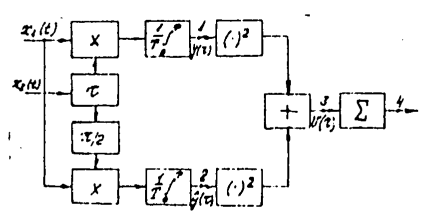

Figure 1

The signal at the output of the multibeam channel is represented in the form [1, 2]:

$$S(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S(\omega)P(\omega,t)\exp(j\omega t)d\omega, \quad (1)$$

where $S(\omega)$ is the spectrum of the emitted signal, $P(\omega,t)$ is the transmission characteristic of the channel;

$$P(\omega,t)=\sum_{n=1}^{N}\alpha_n \exp\{-j\omega[\tau_{no}+\Delta\tau_n(t)+\beta_n t+\frac{\varphi_n}{|\omega|}]\}, \quad (2)$$

where N is the number of beams in the channel, α_n is the transmission coefficient of the channel with respect to the n beam, β_{12} is the doppler coefficient for the n beam, τ_{no} is the constant component of the delay time of the signal in the n beam, $\Delta\tau_n(t)$ are the delay time fluctuations, ϕ_n is the phase shift of the signal in the beam. We shall consider that the antenna insures spatial resolution of the beams so that the following processes will reach the inputs of the correlator:

$$x_1(t)=S_1(t)+n_1(t),$$
$$x_2(t)=S_2(t)+n_2(t), \quad (3)$$

where $S_1(t)$ and $S_2(t)$ are the analyzable signals which are the same initial noise signal arriving at the reception point along different beams, $n_1(t)$ and $n_2(t)$ -- interference against the background of which the signals -- independent stationary processes with zero mathematical expectation and dispersion σ^2_n -- are received. For $\Delta fT>>1$ (Δf is the preselector band) the processes at the outputs of the integrators (Vol 1 and Vol 2) have in practice gaussian distribution. Then for the given probability of false alarm F the probability of proper detection is:

$$D=\int_{\sqrt{2\ln F}}^{\infty} z\exp(-\frac{z^2+\mu^2}{2})I_0(\mu z)dz, \quad (4)$$

where

$$\mu^2=\frac{m^2}{\sigma^2}=\frac{\sqrt{m_1^2+m_2^2}}{\sigma^2}, \quad (5)$$

m_1 and m_2 are the mathematical expectations of the processes in Vol 1 and Vol 2, σ^2 is the dispersion of the processes in Vol 1 and Vol 2. As is obvious from (4) for fixed F, D is a function of the unique parameter μ; therefore the problem of optimizing the integration time reduces to the problem of maximizing the ratio m^2/σ^2 by the parameter T.

When calculating the parameter m let us first carry out averaging of the processes $y(\tau)$ and $\hat{y}(\tau)$ by the fluctuations of the input signals ($M_c\{\cdot\}$), considering the channel parameters fixed, and the averaging with respect to the fluctuations of the channel parameters ($M_k\{\cdot\}$) will be performed after squaring:

$$m^2 = M_\kappa\{v^2(\tau)\} = M_\kappa\{[M_c\{y(\tau)\}]^2\} + M_\kappa\{[M_c\{\hat{y}(\tau)\}]^2\}. \qquad (6)$$

Here we shall consider that the fluctuations of the delay time in each beam are independent gaussian processes with the dispersion σ^2_τ and the correlation coefficient

$$\rho(t_1-t_2) = exp\{-\frac{|t_1-t_2|}{\theta}\} \approx 1 - \frac{|t_1-t_2|}{\theta}, \qquad (7)$$

and the spectral density of the signals $S_1(t)$ and $S_2(t)$ is defined by an expression of the type:

$$F(\omega) = exp\{-\frac{\pi(\omega-\omega_0)^2}{\Delta\omega^2}\}. \qquad (8)$$

Then, using (1) and (2) and considering the substitution $\omega_1\omega_2 \approx \omega_0^2$ admissible, for $\Delta\beta=0$ we obtain:

$$m^2 = \frac{1}{4\pi^2T^2}\cdot\frac{\Delta\omega}{2}exp(-\frac{\alpha^2\Delta\omega^2}{2\pi})\left[\frac{2\omega_0^2\sigma_\tau^2\theta T - \theta^2(1-exp\{-2\omega_0^2\sigma_\tau^2 T/\theta\})}{2\omega_0^4\sigma_\tau^4}\right], \qquad (9)$$

where $\alpha = \Delta\tau_{12} + (1-\beta_2)\tau + \Delta\varphi_{12}/|\omega|$.

The dispersion of the process at the integrator output for $\sigma_n^2 >> \sigma_c^2$ will be determined in practice by the dispersion of the noise component and can be represented in the form:

$$\sigma^2 = \frac{1}{T}B(\tau). \qquad (10)$$

Substituting (10) and (9) in (5) for $\alpha=0$ and investigating μ^2 at the maximum with respect to T, we obtain the expression defining the optimal integration time:

$$T_{opt} \approx \frac{0.75\theta}{\omega_0^2\sigma_\tau^2}.$$

In the case where the delay time fluctuations can be neglected, from (6) we obtain:

$$m^2 = \frac{\Delta\omega^2}{8\pi^2}exp\{-\frac{\Delta\omega^2}{2\pi}[\Delta\tau_{12} + \frac{\Delta\beta_{12}T}{2} + (1-\beta_2)\tau]\}\left[\frac{\sin\omega_0\Delta\beta_{12}T/2}{\omega_0\Delta\beta_{12}T/2}\right]. \qquad (11)$$

Using (10) and (11) at the maximum correlation point $T_{opt}=2/\Delta\beta_{12}\omega_0=a3/\Delta f_d$ where $\Delta f_d=f_0\Delta\beta_{12}$ is the doppler shift of the central frequency. Thus, the integration time in the correlator is limited both by the fluctuations of the signal delay time in the channel and the doppler frequency shift. Here there are grounds for considering that in the central part of the sound range the restriction with respect to the doppler effect is more significant.

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PROCESSING THE SIGNALS OF A SEPARATED GROUP OF RECEIVERS

By V. G. Berkuta, S. V. Pasechnyy, pp 74-80

Let us consider the problem of processing acoustic signals in a nonuniform field by a system of n nondirectional receivers separated in space (see Fig 1). Let us propose that the interference $N(t, \vec{x})$ is gaussian and uncorrelated between the individual receivers. In the beam approximation the signals arriving at the reception point $\vec{x}_\ell$, considering the propagation in the medium, possible reflection from the boundaries and the target, will be represented in the form:

$$S_\ell(t) = \sum_{i=1}^{m_1} \sum_{k=1}^{m_2} \varepsilon_i \zeta_{k\ell} S_i T_k(\theta,\varphi) R_{k\ell} \cdot A(t+\tau_i+\tau_{k\ell}) \cos[\omega_0(t+\tau_i+\tau_{k\ell}) + \Psi(t+\tau_i+\tau_{k\ell}) + \varphi_i + \varphi_{k\ell}]. \quad (I)$$

Here m_1 and m_2 are the number of beams arriving at the reflecting target and at the point $\vec{x}_\ell$, respectively; ε_i, ζ are the coefficients characterizing the fluctuations of the amplitudes of the individual beams occurring on reflection of sound from the wavy surface of the ocean, transmission of it through the nonuniform layers and with random displacement of the reflecting target; S_i, $R_{k\ell}$ are the amplitudes of the individual beams considering the damping of the sound in the medium and the focusing factor; $T_k(\theta,\phi)$ is the coefficient taking into account the reflection from the target with the scattering indicatrix $K(\theta,\phi)$; τ_i, $\tau_{k\ell}$, ϕ_i, $\phi_{k\ell}$ are the delay times and the random initial phases of the individual beams respectively; $A(t)$, ω_0, $\Psi(t)$ are the parameters of the emitted signal. The algorithmof the optimal spatial group processing considering [1] has the form:

$$U_0 = \sum_{\ell=1}^{n} \sum_{j=1}^{n} \int_{-(\tau_i+\tau_k)}^{T-(\tau_i+\tau_k)} \int U_\ell(t) U_j(t') H_{\ell j}(t,t') \, dt \, dt', \quad (2)$$

where $U_\ell(t) = S_\ell(t) + N(t)$ is the adopted oscillation on the ℓ-th receiver; H_{ℓ_j} is the solution of the systemof integral equations:

$$\sum_{j=1}^{n}\int_{-(\tau_i+\tau_\kappa)}^{T-(\tau_i+\tau_\kappa)}[K_{N_{\ell j}}(t,t')+K_{S_{\ell j}}(t,t')]H_{jh}(t',\tau)dt'=L_{\ell h}(t,\tau), \quad (3)$$

$$\sum_{j=1}^{n}\int_{-(\tau_i+\tau_\kappa)}^{T-(\tau_i+\tau_\kappa)}L_{\ell j}(t,t')K_{N_{jh}}(t',\tau)dt'=K_{S_{\ell h}}(t,\tau). \quad (4)$$

Here $K_{S_{\ell j}}(t,t')$ and are the time-space correlation functions of the signal and the noise respectively.

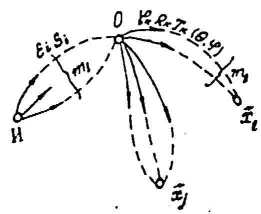

Figure 1

The expressions for $K_{S_{\ell j}}(t,t')$ and $K_{N_{\ell j}}(t,t')$ considering (1) have the form:

$$K_{S_{\ell j}}(t,t')=\delta_{\ell j}\sum_{i=1}^{m_1}\sum_{\kappa=1}^{m_2}S_i^2T_\kappa(\theta,\varphi)R_{\kappa\ell}^2[s(t+\tau_i+\tau_{\kappa\ell})\times s(t'+\tau_i+\tau_{\kappa\ell})+\hat{s}(t+\tau_i+\tau_{\kappa\ell})\hat{s}(t'+\tau_i+\tau_{\kappa\ell})] \quad (5)$$

and

$$K_{N_{\ell j}}(t,t')=\delta_{\ell j}K_{N_\ell}(t,t'), \quad (6)$$

where $\delta_{\ell j}$ is the Kronecker symbol; $s(t)=A(t)\cos[\omega_0t+\Psi(t)]$ and $\hat{s}(t)=A(t)\sin[\omega_0t+\Psi(t)]$ is the emitted signal and the signal conjugate to it according to Hilbert, respectively. Substituting (5), (6) in (3), (4), after transformations we obtain the structure of the optimal receiver in the form:

$$U_0=\sum_{\ell=1}^{n}\left\{\sum_{i=1}^{m_1}\sum_{\kappa=1}^{m_2}\frac{S_i^2T_\kappa^2(\theta,\varphi)R_{\kappa\ell}^2\sigma_{N\ell}^{-2}}{1+\sum_{i=1}^{m_1}\sum_{\kappa=1}^{m_2}S_i^2T_\kappa^2(\theta,\varphi)R_{\kappa\ell}^2\sigma_{N\ell}^{-2}\int_0^T s(t)\delta(t)dt}\times\left|\int_0^T U_\ell(t-\tau_i-\tau_{\kappa\ell})\delta(t)dt\right|^2\right\}-\sum_{\ell=1}^{n}\left\{\sum_{i=1}^{m_1}\sum_{\kappa=1}^{m_2}\gamma_{i\kappa}^{\ell}\left|\int_0^T U_\ell(t-\tau_i-\tau_{\kappa\ell})\delta(t)d(t)\right|^2\right\}. \quad (7)$$

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The structural diagram of the receiver (7) is presented in Fig 2. In particular, when the multibeam nature of the propagation can be neglected, from (7) we have the result presented in [2], and in the case of matching the reception and emission points, the result obtained in [3]. Under actual conditions when the values of the delays τ_i, $\tau_{k\ell}$ and the signal/noise ratio $S_i^2 T_k(\theta,\varphi) R^2_{k\ell}/\sigma^2_{n_k}$, quasioptimal algorithms are used, for example, with the cumulative decision-making rule. The group processing algorithm with cumulative decision-making rule on reception of multibeam signals by separated nondirectional receivers has the form:

$$U_0 = \left\{\sum_{\ell=1}^{n}\left[\operatorname{sgn}\left(\sum_{i=r}^{r+q}\operatorname{sgn}(y_{i\ell}-\Pi_{i\ell})-P_\ell\right)\right]-K\right\}. \qquad (8)$$

where $y_{i\ell}$ is the effect at the output of the detector of the optimal factor of the ℓ-th receiver at the r-th point in time (see Fig 3); $\Pi_{i\ell}$, P_ℓ, K are the threshold levels; q is the interval of sliding summation.

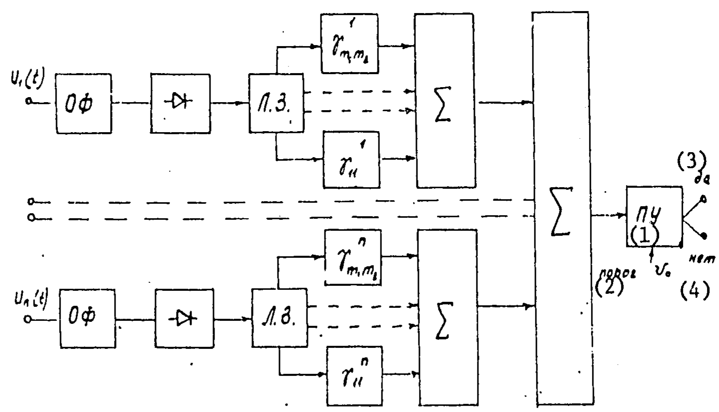

Figure 2

Key:

1. Receiver
2. Threshold
3. Yes
4. No

The structural diagram of the channel with the cumulative decision rule "k out of n" is presented in Fig 3. The threshold levels $\Pi_{i\ell}$ are established in accordance with the adopted cumulative rule in each receiver "p out of g" where g is the number of investigated alternatives. In the adder of each receiver the sequences of 1's and 0's are accumulated in the interval q which is proportional to the maximum delay between the beams arriving at the given receiver. If the number of 1's in some interval q exceeds the threshold p, then a 1 reaches the final adder. On arrival of k ones at the input of the adder with channels, the receiver makes the decision of the presence of a target.

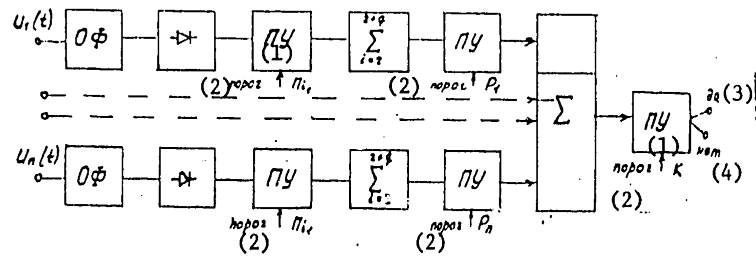

Figure 3

Key:

1. Receiver
2. Threshold
3. Yes
4. No

Let us investigate the noiseproofness of the channels for optimal and quasioptimal group processing with cumulative decision rule. Let us propose that the medium is uniform and isotropic. Then, according to [2], the probability of proper detection of the system is:

$$D\{U_0\} = \prod_{\ell=1}^{n} C_\ell^{\beta_\ell} i^{n+3} \sum_{j=1}^{n} \frac{1}{(\beta_j-1)!} \left[\frac{d^{\beta_j-1}}{d\xi^{\beta_j-1}} \left(\frac{e^{-i\xi v_0}}{i\xi \prod_{\ell=1,\ \ell\ne j}^{n} (\xi + iC_\ell)^{\beta_\ell}} \right) \Bigg|_{\xi=-iC_j} \right], \quad (9)$$

where $C_\ell=1/2\mu_\ell$; μ_ℓ is the signal/noise ratio at the output of the ℓ-th channel of the receiving channel; β_ℓ is the degree of multiplicity of the receivers with identical signal/noise ratio μ_ℓ, $\sum_\ell \beta_\ell=n$. The probability of false alarm will be described by the expression analogous to (9), but with other coefficients $B_\ell=(1+\mu_\ell)/2\mu_\ell$. The probability of proper detection of the quasioptimal receiver is determined by the cumulative probability that the threshold will be exceeded in the n receivers no less than k times:

$$D\{K,\Pi\} = \sum_{\ell=K}^{n} \frac{1}{\ell!} \left\{ \frac{d^\ell \varphi_n(x)}{dx^\ell}\Big|_{x=0} \right\}, \quad (10)$$

where $\varphi_n(x)=\prod_{\ell=1}^{n}(1-p_\ell+p_\ell x)$ is the derivative function; $p_\ell = e^{-\Pi^2/2(\mu_\ell+1)}$

is the probability that the threshold will be exceeded in the ℓ-th receiver; $\Pi=\Pi_\ell/\sigma_{n\ell}$ is the normalized threshold level. Since the probability of false alarm in each channel is constant, the probability of false alarm of the system is:

$$F\{K,\Pi\} = \sum_{\ell=K}^{n} C_n^\ell e^{-\frac{\Pi^2 \ell}{2}} (1-e^{-\Pi^2/2})^{n-\ell}. \quad (11)$$

By formulas (9-11), calculations were made of the normalized detection zones of the separated systems. The normalization was carried out with respect to a matched medium with the same acoustic parameters as the separated system, which in the calculations made it possible to exclude such parameters as the emitted power, the speed of sound, the volumetric scattering coefficient, and so on.

A study was made of the following separated systems:

One emitter and n nondirectional receivers (n=3,5,8) arranged in a circle and operating in a uniform medium (Figures 4-6);

One emitter and n receivers that are directional in the vertical plane (n=3,5,8) arranged in a circle and operating in a uniform medium;

One emitter and five directional receivers arranged in a circle and operating in a nonuniform medium with standard c-profile taken from [4] (see Fig 8).

A study is made of the problems of the operation of the separated system against a background of an additive mixture of noise and reverberation interference; comparison of the optimal and cumulative algorithms for group-processing; selection of the optimal cumulative rule; effect of the directionalness of the receivers in the vertical plane; consideration of the layered nonuniformity of the medium. The parameter for calculating the normalized detection zones was the value of $\beta_0=[\sigma^2_{noise}/\sigma^2_{rev0}]_{out}$, where $[\sigma^2_{rev0}]_{out}$ is the dispersion of the reverberation interference at the output of the matched medium at maximum range.

As a result of the studies it is possible to draw the following conclusions:

The effectiveness of the group processing in the separated system essentially depends on the ratio of the noise and reverberation interference where the effectiveness decreases on predominance of the noise of reverberation (Fig 4);

For the predominant reverberation interference the area of the normalized detection zone in optimal group processing is three times the total area of the detection zone for the system with matched media, and with cumulative group processing, it is twice the total area of the detection zone for the system with matched media (see Figures 5-6);

The area of the normalized detection zone with predominant reverberation interference is in practice proportional to the number of receivers. In contrast to the indicated situation for predominant noise the increase in the number of receivers is ineffective (Figures 4 to 6);

The optimal cumulative rule depends on the number of receivers in the system and the nature of the acting interference (Fig 7);

The directionalness of the receivers in the vertical plane in practice has no influence on the effectiveness of the group processing;

The normalized detection zone on various levels under the conditions of the layered-nonuniform medium undergoes significant variations (see Fig 9).

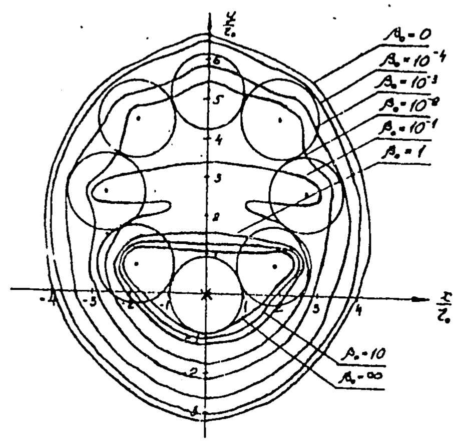

Figure 4. Normalized detection zone of a separated system with optimal processing

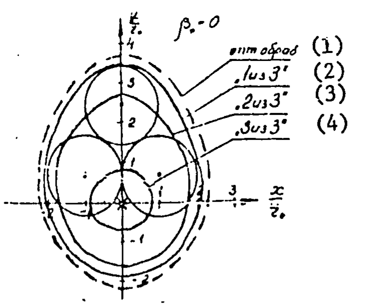

Figure 5. Normalized detection zone for reverberation interference
Key:
1. optimal processing
2. 1 out of 3
3. 2 out of 3
4. 3 out of 3

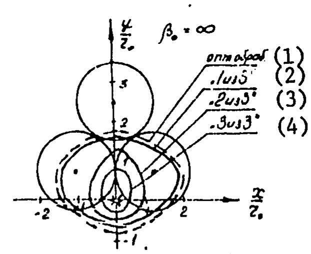

Figure 6. Normalized detection zone for noise interference
Key:
1. optimal processing
2. 1 out of 3
3. 2 out of 3
4. 3 out of 3

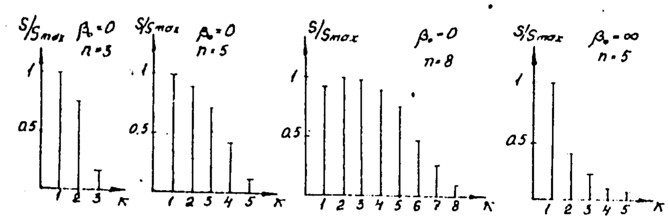

Figure 7. Distribution of the normalized detection zone areas during cumulative processing of the k out of n type

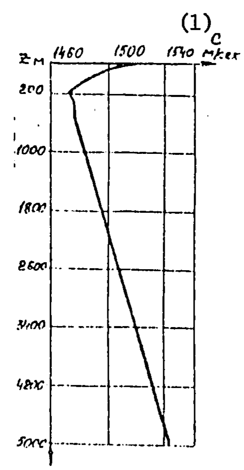

Figure 8. Sound velocity profile.
Key:
1. m/sec

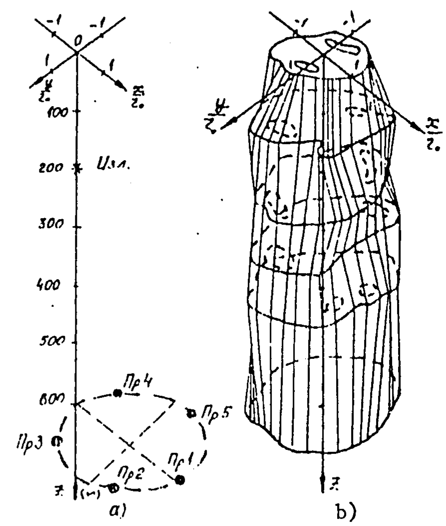

Figure 9. Normalized detection zone of a separated system in a layered nonuniform medium:
a) schematic of the arrangement of the receiver and the emitter;
b) normalized detection zone for predominant noise interference

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INDETERMINACY FUNCTION FOR PROCESSING SIGNALS BY A GROUP OF SEPARATED RECEIVERS

By S. V. Pasechnyy, S. V. Skripchenko, pp 80-84

Let us investigate the properties of the indeterminacy function as applied to the case of processing signals by a group of receivers separated in space. In accordance with [1] the space-time indeterminacy function can be represented in the form:

$$\Psi(\vec{\lambda},\vec{\lambda}_0)=\int\limits_{\Omega}\int\limits_{t_0(\vec{r})}^{t_0(\vec{r})+T} S(t,\vec{r},\vec{\lambda}_0)B(t,\vec{r},\vec{\lambda})\,dt\,d\vec{r} \Big/ \int\limits_{\Omega}\int\limits_{t_0(\vec{r})}^{t_0(\vec{r})+T} S(t,\vec{r},\vec{\lambda}_0)B(t,\vec{r},\vec{\lambda})\,dt\,d\vec{r}, \quad (1)$$

where $\vec{\lambda}$ is the vector of the proposed coordinates of the signal source to which the receiver is "tuned" and $\vec{\lambda}_0$ is the vector of the two coordinates of the signal source, B(t, $\vec{r}$, $\vec{\lambda}$) is the structure of the optimal receiver calculated for reception at the point $\vec{r}$ of the signal with the parameters $\vec{\lambda}$.

Depending on the selected coordinate system the vector component $\vec{\lambda}$ can have different meaning, for example, in the cartesian system these are the coordinates x, y, z, and in the spherical system, the angles α and γ and the range r. Let us note that in contrast to the cases of measuring the coordinates by the joint system in separated systems, the origin of the coordinates cannot coincide with one of the system receivers.

Let us consider the group of nondirectional receivers separated in space at a distance significantly exceeding the interference correlation interval. As was demonstrated in [2], the direction and the output of the optimal receiver on detection of a signal with random phase and fluctuating amplitude $S_i(t)=\varepsilon_i S_i S_0(t,\varphi)$ in this case is described by the expression of the type:

$$U_{\text{вых}(1)}=\sum_{i=1}^{n}\frac{\gamma_i}{S_i^2}\Big\{\Big[\int_0^T u_i(t-\tau_{i0})B(t-\tau_i)\,dt\Big]^2+\Big[\int_0^T u_i(t-\tau_{i0})\hat{B}(t-\tau_i)\,dt\Big]^2\Big\}, \quad (2)$$

Key: 1. out

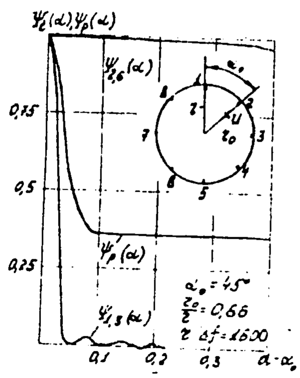

Figure 1

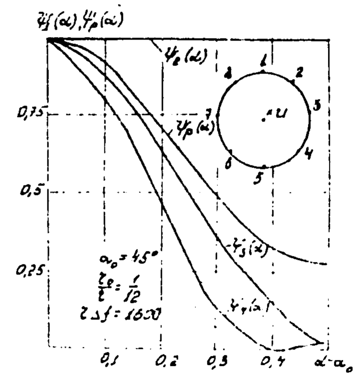

Figure 2

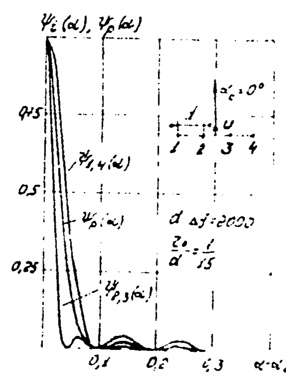

Figure 3

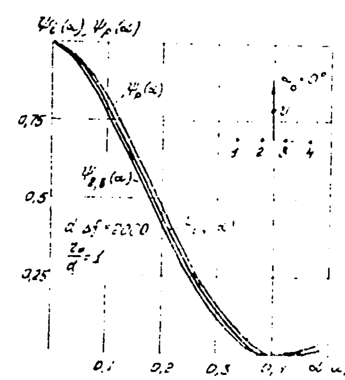

Figure 4

where n is the number of receivers, $\gamma_i = \frac{\mu_i^2}{(1+\mu_i d_0)}$ is the weight coefficient defined by the signal/noise ratio $\mu_i = S_i^2/\sigma_N^2$ in the i-th receiver, $d_0 = \int_0^T S_0(t)B(t)dt$, $S_0(t)$ is a copy of the signal, B(t) is the structure of the receiver obtained as a solution to the system of integral equations with the kernel $\rho_N(t,t_1)$ -- the correlation function of the interference -- the index ^ denotes the Hilbert transformation.

The signal component of the output effect (2) as a result of the amplitude fluctuations of the input signal is a random variable. When analyzing the properties of the signals and the receiving channels it is expedient to consider its mean value:

$$\overline{U}_{cu2} = \sum_{i=1}^{n}\gamma_i\Big\{\overline{\Big[\int_0^T \varepsilon_i S_0[(t-\tau(\vec{\lambda}_0,\vec{z}_i)),\varphi]B(t-\tau(\vec{\lambda},\vec{z}_i))dt\Big]^2} + \overline{\Big[\int_0^T \varepsilon_i S_0[(t-\tau(\vec{\lambda}_0,\vec{z}_i)),\varphi]\hat{B}(t-\tau(\vec{\lambda},\vec{z}_i))dt\Big]^2}\Big\} = \bar{\varepsilon}^2/d|^2\sum_{i=1}^{n}\gamma_i\Psi_i(\vec{\lambda},\vec{\lambda}_0,\vec{z}_i). \quad (3)$$

where $\Psi_i(\vec{\lambda},\vec{\lambda}_0, \vec{r}_i)$ is the indeterminacy function by the parameter $\vec{\lambda}$ in the i-th receiver $|d|^2=\{[\int_0^T S_0(t,\tau_0,\varphi)\mathcal{B}(t,\tau_0)dt]^2+[\int_0^T \dot{S}_0(t,\tau_0,\varphi)\hat{\mathcal{B}}(t,\tau_0)dt]^2\}$.

Normalizing expression (3) by its maximum value, we obtain the desired indeterminacy function:

$$\Psi_p(\vec{\lambda},\vec{\lambda}_0)=\sum_{i=1}^{K}\gamma_i\Psi_i(\vec{\lambda},\vec{\lambda}_0,\vec{r}_i)\Big/\sum_{i=1}^{n}\gamma_i . \qquad (4)$$

Thus, the indeterminacy function of a group of directional receivers separated in space is the sum of the indeterminacy functions of the individual receivers weighted with the weight coefficient defined by the signal/noise ratio and normalized by the sum of the weight coefficients. The form of the function $\Psi_p(\vec{\lambda},\vec{\lambda}_0)$ depends not only on the signal and interference properties but also on the number and mutual location of the receivers and the coordinates of the point at which the signal source is located.

As an example, in Figures 1 to 4, graphs are presented for the indeterminacy functions $\Psi_p(\vec{\lambda},\vec{\lambda}_0)$ and $\Psi_i(\vec{\lambda},\vec{\lambda}_0)$ by the azimuth α when using the wide band signals for two models of the systems: with location of the receivers around a circle and along a line. The position of the signal source in the figures is denoted by an "asterisk."

As is obvious from these figures, the group of nondirectional receivers has very high resolution with respect to the angular coordinates, which increases with an increase in the variation rate of the signal delay time in the receivers closest to the signal source. It is also necessary to note that the resolution with respect to angle for the group of receivers depends to a significant degree on the type of its indeterminacy function with respect to time.

In conclusion let us consider the peculiarities of constructing the precision characteristics of the estimates of the coordinates $\vec{\lambda}$ when using the group processing method. As is known [3], the accuracy of estimating the signal parameters is determined by the covariance matrix of the errors Σ, the elements of which can be calculated using the Fisher information matrix B:

$$\Sigma_{kj}=B_{kj}^{-1} ; \quad B_{kj}=-\left\langle\frac{\partial^2}{\partial\lambda_k\partial\lambda_j}\ln p(u/\vec{\lambda})\right\rangle\Big|_{\vec{\lambda}=\vec{\lambda}_0} , \qquad (5)$$

where B_{kj}^{-1} are the elements of the matrix inverse to B, $p(u/\vec{\lambda})$ is the plausibility function.

It is possible to demonstrate that for the investigated signal processing channel

$$B_{kj}\approx-\frac{\partial^2}{\partial\lambda_k\partial\lambda_j}\overline{U}_{cu2}=-\sum_{i=1}^{n}\overline{\varepsilon^2}|d|^2\gamma_i\frac{\partial^2}{\partial\lambda_k\partial\lambda_j}\Psi_i(\vec{\lambda},\vec{\lambda}_0,\vec{r}_i)\Big|_{\vec{\lambda}=\vec{\lambda}_0} . \qquad (6)$$

Inasmuch as the measured parameters $\vec{\lambda}$ are uniquely expressed in terms of the signal delay time r, using the differentiation rules of the complex functions (6) can be represented in the form:

$$B_{kj} = \sum_{i=1}^{n} \left(-\bar{\varepsilon}^2 \gamma_i |\alpha|^2 \frac{\partial^2 \psi_i(\tau)}{\partial \tau^2}\right) \frac{\partial \tau_i}{\partial \lambda_k} \frac{\partial \tau_i}{\partial \lambda_j} = \sum_{i=1}^{n} B_{\tau\tau_i} \frac{\partial \tau_i}{\partial \lambda_k} \frac{\partial \tau_i}{\partial \lambda_j} , \qquad (7)$$

where $B_{\tau\tau_i}$ is an element of matrix inverse to the error matrix for measuring the time parameters of the signal in the i-th receiver.

Expression (7) indicates the relation between the accuracy of measuring the time parameters of the signals and the spatial coordinates of the signal source. This relation can be more clearly demonstrated in the case where the only unknown parameter of the signal in the i-th receiver is the delay time and only one coordinate of the source is subject to measurement. In this case:

$$B_{\tau\tau_i} = \sigma_{\tau_i}^{-2} ; \quad \sigma_{jj}^2 = B_{jj}^{-1} = 1 / \sum_{i=1}^{n} \sigma_{\tau_i}^{-2} \left(\frac{\partial \tau_i}{\partial \lambda_j}\right)^2 , \qquad (8)$$

where $\sigma_{\tau_i}^2$ is the dispersion of the signal delay time estimate in the i-th receiver.

Thus, the potential accuracy in measuring the coordinates of the signal source by a group of nondirectional receivers is determined by the accuracy of measuring the time parameters of the signals in the receivers and the type of functional relation of the signal parameters and coordinates of the receivers and the signal source. In order to increase the resolution and accuracy of the measurement it is desirable to have available receivers such that in the receivers closest to the signal source the variation of the signal delay time when measuring the coordinates $\vec{\lambda}$ will be maximal.

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DIRECTION FINDING OF A NOISE SOURCE IN A RANDOM NONUNIFORM MEDIUM

By K. Yu. Agranovskiy, Yu. A. Kartashov, Ye. B. Kudashev, L. A. Reshetov, pp 84-86

In this paper a study is made of the accuracy of the effective estimates of the angular coordinates of the random gaussian field against a background of gaussian interference. Primary attention is given to the model of the acoustic field passing through a turbulent medium. Let us consider the uniform stationary random field $\xi(t,\vec{r})$ which is displaced in the direction of the transfer vector $\vec{U}=(|\vec{U}|\cos\lambda,\ |\vec{U}|\cos\theta,\ |\vec{U}|\cos\nu)$ where λ, θ, and ν are the direction cosines, where $\cos^2\lambda+\cos^2\theta+\cos^2\nu=1$. We shall consider that the investigated field is "frozen." Then the wave-frequency spectrum of the random field $\Phi(\omega,\vec{\chi})$ will be equal to $\delta(\vec{\chi}\vec{U}^T-\omega)\Phi(\vec{\chi})$, where $\vec{\chi}=(\chi_1,\chi_2,\chi_3)$ is the row vector of the wave numbers and $\vec{U}^T$ is the transposed matrix.

In order to find the mutual spectrum $W(\omega,\vec{R}_{\ell k})$ and the mutual correlation of the signals $K(\tau,\vec{R}_{\ell k})$ at the output of the ℓ-th and k-th nondirectional hydrophones with unit sensitivity, let us write the radius vector joining these receivers in the form:

$$\vec{R}_{\ell k}=(h_\ell\cos\gamma_\ell-h_k\cos\gamma_k,\ h_\ell\cos\alpha_\ell-h_k\cos\alpha_k,\ h_\ell\cos\beta_\ell-h_k\cos\beta_k), \quad (1)$$

where h_ℓ, h_k is the distance from the geometric center of the system to the ℓ-th and the k-th hydrophones, $\cos\gamma$, $\cos\alpha$, $\cos\beta$ are the direction cosines. By definition the mutual spectrum is equal to:

$$W(\omega,\vec{R}_{\ell k})=\iiint_{-\infty}^{\infty}\Phi(\omega,\vec{æ})e^{-j\vec{æ}\vec{R}^T_{\ell k}}dæ_1dæ_2dæ_3 \quad (2)$$

Let, for example, $\vec{U}(\chi_1,0,0)$. Then:

$$W(\omega,\vec{R}_{\ell k})=\frac{1}{U}\exp j\left\{\frac{\omega}{|\vec{U}|}[h_\ell\cos\gamma_\ell-h_k\cos\gamma_k]\right\}\iint_{-\infty}^{\infty}\Phi\left(-\frac{\omega}{|\vec{U}|},æ_2,æ_3\right)\times \quad (3)$$
$$\times\exp j\{æ_2[h_\ell\cos\alpha_\ell-h_k\cos\alpha_k]+æ_3[h_\ell\cos\beta_\ell-h_k\cos\beta_k]\}dæ_2dæ_3.$$

Frequently the wave spectrum of the real fields is subject to factorization, that is,

$$\Phi(\varkappa_1, \varkappa_2, \varkappa_3) = \sum_{n=1}^{Q} C_n \Phi_{1n}(\varkappa_1)\Phi_{2n}(\varkappa_2)\Phi_{3n}(\varkappa_3), \qquad (4)$$

where C_n are the constants. Substituting (4) in (3), we obtain:

$$W(\omega, \vec{R}_{\ell\kappa}) = \frac{1}{|\vec{U}|} \exp -j\left\{\frac{\omega}{|\vec{U}|}[h_\ell \cos\gamma_\ell - h_\kappa \cos\gamma_\kappa]\right\} \times$$
$$\times \sum_{n=1}^{Q} C_n \Phi_{1n}\left(\frac{\omega}{|\vec{U}|}\right) B_{2n}(h_\ell \cos\alpha_\ell - h_\kappa \cos\alpha_\kappa) B_{3n}(h_\ell \cos\beta_\ell - h_\kappa \cos\beta_\kappa),$$

where $B_{2n}(\cdot)$ and $B_{3n}(\cdot)$ is the result of the Fourier transformation of the function $\Phi_{2n}(\cdot)$ and $\Phi_{3n}(\cdot)$, respectively. Let the antenna have N nondirectional hydrophones. The covariation matrix of errors of the effective estimates of the vector coordinates of the parameters $\vec{\alpha}=(\alpha_1,\ldots,\alpha_M)$ of the random field can be obtained by inversion of the information matrix $I_{pm}(p, m=1,\ldots,M)$ [1].

As an example, let us consider the reception of the signal field on the antenna made up of two hydrophones located at the distance d. Let us find the value that is the inverse of the dispersion of the estimate:

$$D_\alpha^{-1} = I_{11} = \frac{2T\Delta\omega}{\pi} \cdot \frac{q^2}{(1+q)^2 - q^2 B_2^2(d\cos\alpha)} \left\{\left[\frac{\partial B_2(d\cos\alpha)}{\partial\alpha}\right]^2 + \right.$$
$$\left. + g\left[\frac{d}{c}\cos\alpha\, B_2(d\cos\alpha)\right]^2 + 2q^2 \frac{\left[B_2(d\cos\alpha)\frac{\partial B_2(d\cos\alpha)}{\partial\alpha}\right]^2}{(1+q)^2 - q^2 B_2^2(d\cos\alpha)}\right\},$$

T is the analysis time, $q=S_0/N_0$ is the spectral signal/noise ratio, $c=|\vec{U}|$ is the speed of sound, $q=\Delta\omega^2/3$ for the wide band process and $q=\omega_0^2$ for the narrow band process.

Let us find the magnitude of the decrease in accuracy of the direction finding on the appearance of a nonuniformity on the wave front:

$$\rho = \frac{1+2q+q^2[1-B_2^2(d)]}{(1+2q)\; B_2^2(d)}, \qquad \alpha = 0.$$

It is obvious that the decorrelation on the front always leads to an increase in the dispersion of the estimate of the angular position of the source. The losses in accuracy increase with an increase in the sizes of the antenna and with an increase in signal/noise ratio.

One of the actual factors leading to the formation of decorrelation at the wave front is turbulence of the ocean. Let us assume that the random field is the result of the effect of a plane harmonic wave with frequency ω_0 on a turbulent layer of thickness L with a gaussian correlation coefficient of the fluctuations of the index of refraction. For the calculations the correlation coefficient of the sound field at the output of the turbulent layer in the direction coinciding with the axis of the wave numbers χ_2 is borrowed from [2]. The standard magnitude of the mean square of the pulsations of the index of refraction for marine turbulence has an order of 10^{-8} to 10^{12}. Then for routes 500 to 1000 meters long and a signal having a central frequency f_0=10 kilohertz, the magnitude of the decorrelation at the wave front as a result of the scattered field is 0.8 to 0.99.

Analyzing the relations obtained, it is possible to draw the following conclusions:

If the random field is observed with correlation coefficient decreasing at infinite $B_2(d)$, then the increase in the antenna dimensions does not always lead to a decrease in the dispersion of the estimate;

If the antenna dimensions and signal/noise ratio are small, the dependence of the dispersion estimate on the angle α is close to $(\cos \alpha)^{-2}$;

For large size antennas, a sharp decrease in the dispersion is possible for large angles α. This is explained by the fact that in the absence of noise ($q \to \infty$) the determinant of the spectral matrix is equal to zero for $\alpha=90°$, the case of the so-called "singular" estimate where in a finite analysis space-time interval it is possible to obtain infinitely high accuracy. For $q<\infty$ the accuracy of the estimates is always finite, but the nature of the dependence is maintained.

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SYNTHESIS OF AN ASYMPTOTICALLY OPTIMAL DISCRMINATOR FOR DETERMINING THE COORDINATES OF THE WAVE SOURCE IN THE PRESENCE OF "MIRROR" RE-REFLECTION

By L. A. Bespalov, L. G. Goverdovskaya, A. M. Derzhavin, L. A. Reshetov, A. G. Strochilo, pp 86-89

The device for determining the coordinates of the noise signal source is synthesized under the condition that the signal field from the radiator (И) is the sum of two plane waves, one of which (I) is propagated along the shortest path to the geometric center of the antenna (0), and the other (II) is reflected from the "mirror" point. It is proposed that the antenna is made up of N nondirectional point receivers with identical unit sensitivity. The placement of the receivers on the plane can be arbitrary. The propagation rate of the sound in the medium is constant. Let the investigated process have normal distribution with zero mean, a continuous correlation function $K(t,s)\in C_{[0,T]\times[0,T]}$, where T is the length of the realization, and the interference is the internal noise of the receivers described by the model of "white" normal stationary noise with spectral density N_0. The logarithm of the plausibility function in the investigated case has the form [1]:

$$\ln W[X(t),\alpha] = \ln Q(\alpha) - \frac{1}{2}\int_0^T\int_0^T X'(t)R(t,s,\alpha)X(s)\,dt\,ds, \qquad (1)$$

X(s) is the column matrix of the realizations at the output of the antenna, X'(t) is the transposed matrix, $R(t,s,\alpha)$ is the quadratic matrix of dimension NxN, which is the solution of the integral equation:

$$\int_0^T B(y,t,\alpha)R(t,s,\alpha)\,dt = \delta(y-s)E, \qquad (2)$$

where $B(y,t,\alpha)$ is the covariation matrix of the N-dimensional gaussian random process at the output of the antenna system, $\alpha=(\alpha_1,\ldots,\alpha_\ell,\ldots,\alpha_M)$ is the vector of the desired parameters, E is a unit matrix. In our case the matrix $B(\cdot)$ is made up of delta and continuous components:

$$B(y,t,\alpha) = N_0\delta(y-t)E + K(y,t,\alpha), \qquad (3)$$

where the matrix $K(y,t,\alpha)=\|K_{pk}(y,t,\alpha)\|$, the elements of which are mutual correlations of the signals of the p-th and k-th receivers and they are distinguished from the autocorrelation functions only by the shift of the corresponding time arguments. It is known [2] that the solution of the integral equation (2) for $B(y,t,\alpha)$ defined by the equality (3) is the sum of the matrices

$$R(t,s,\alpha)=\frac{1}{N_0}E[\delta(t-s)-h(t,s,\alpha)], \tag{4}$$

where $h(t,s,\alpha)$ is a continuous function in the square $[0.T]\times[0,T]$. It is easy to show that $h(\cdot)$ satisfies the integral equation

$$h(t,s,\alpha)-\frac{K(t,s,\alpha)}{N_0}=-\frac{1}{N_0}\int_0^T h(t,z,\alpha)K(z,s,\alpha)dz. \tag{5}$$

For construction of the structure of the estimate and calculation of its limiting accuracy it is necessary to introduce additional restrictions on the form of the solution $h(\cdot)$, namely:

$$G_{pk\ell}(t,s,\alpha)=\frac{\partial h_{pk}}{\partial\alpha_\ell}\in L^2[0,T]\times[0,T] \tag{6}$$

for any α_ℓ from the region of definition of the vector of the parameters and $\ell=1,\ldots,M$; $p,k=1,\ldots,N$. Let us calculate the derivative of the plausibility functional by the correaponding coordinate of the vector of the parameters. According to [1], [3]:

$$\frac{\partial \ln Q(\alpha)}{\partial\alpha_\ell}=-\frac{1}{2}\sum_{p=1}^{N}\sum_{k=1}^{N}\int_0^T\int_0^T\frac{\partial B_{pk}(t,s,\alpha)}{\partial\alpha_\ell}R_{pk}(t,s,\alpha)dtds,\quad \ell=1,\ldots M. \tag{7}$$

Assuming the unimodal nature of the plausibility function, from the system of plausibility equations averaged with respect to the realization, we obtain:

$$\sum_{p=1}^{N}\sum_{k=1}^{N}\int_0^T\int_0^T\frac{\partial B_{pk}(t,s,\alpha)}{\partial\alpha_\ell}R_{pk}(t,s,\alpha)dtds=-\sum_{p=1}^{N}\sum_{k=1}^{N}\int_0^T\int_0^T B_{pk}(t,s,\alpha)\times\frac{\partial R_{pk}(t,s,\alpha)}{\partial\alpha_\ell}dtds,\quad \ell=1,\ldots,M,\ \alpha=\alpha_0. \tag{8}$$

Using the formulas (4), (7) and (8), we finally have:

$$L_\ell[X(t),\alpha_0]=\frac{\partial \ln W[X(t),\alpha_0]}{\partial\alpha_\ell}=\frac{1}{2N_0}\sum_{p=1}^{N}\sum_{k=1}^{N}\int_0^T\int_0^T G_{pk\ell}(t,s,\alpha_0)\times[x_p(t)x_k(s)-B_{pk}(t,s,\alpha_0)]dtds. \tag{9}$$

Formula (9) generalizes the results of [4] to the case of N>1.

In order to obtain an estimate of the maximum plausibility, it is necessary to find the maximum of the plausibility function or to try to solve the plausibility equation where the former and the latter are difficult to execute. However, if the deviation of α from the known value of α_0 is not great, it is possible to find approximate estimates of the maximum plausibility asymptotically optimal for $\alpha \to \alpha_0$. Actually, assuming that the fluctuation part of the plausibility function is small by comparison with its mathematical expectation, we shall expand $\ln W[X(t),\alpha]$ as a function of M variables in a Taylor series with respect to the powers of $\Delta\alpha_\ell$ and we shall limit ourselves to the first three terms of this expansion:

$$\ln W[X(t),\alpha] = \ln W[X(t),\alpha_0] + \sum_{i=1}^{2} \frac{1}{i!}\left(\frac{\partial}{\partial\alpha_1}\Delta\alpha_1 + \dots + \frac{\partial}{\partial\alpha_\ell}\Delta\alpha_\ell + \dots + \frac{\partial}{\partial\alpha_M}\Delta\alpha_M\right)^i \ln W[X(t),\alpha_0]. \quad (10)$$

Differentiating $\ln W(\cdot)$ with respect to the coordinate α_ℓ and equating the derivatives to zero, we obtain the system of equations:

$$\sum_{q=1}^{M} \Delta\alpha_q \frac{\partial^2 \ln W[X(t),\alpha_0]}{\partial\alpha_\ell\,\partial\alpha_q} = -L_\ell[X(t),\alpha_0];\quad \ell=1,\dots,M. \quad (11)$$

The second-order derivatives usually can be replaced by their mathematical expectations [5], for the dispersion of these random variables is small. Then, solving the system of equations (11), we obtain the estimates of the increments of the parameters

$$\Delta\hat{\alpha}_q = \frac{\Delta_q[X(t)]}{\Delta}, \quad (12)$$

where

$$\Delta = \begin{vmatrix} a_{11}\dots a_{1q}\dots a_{1M} \\ a_{\ell 1}\dots a_{\ell q}\dots a_{\ell M} \\ a_{M1}\dots a_{Mq}\dots a_{MM} \end{vmatrix}, \quad \Delta_q[X(t)] = \begin{vmatrix} a_{11}\dots -L_1(\cdot)\dots a_{1M} \\ a_{\ell 1}\dots -L_\ell(\cdot)\dots a_{\ell M} \\ a_{M1}\dots -L_M(\cdot)\dots a_{MM} \end{vmatrix}, \quad (13)$$

$$a_{\ell q} = \left\langle \frac{\partial^2 \ln W[X(t),\alpha_0]}{\partial\alpha_\ell\,\partial\alpha_q} \right\rangle; \quad \ell, q = 1,\dots,M.$$

The estimates $\Delta\hat{\alpha}_q$ unbiased within the limits of the linear expansion $B_{pk}(\cdot)$ with respect to α_ℓ and for $\alpha=\alpha_0$ coincide with the maximum plausibility estimate. Using the estimates obtained, it is possible to find the angular positions of M on stationary noise sources. In the general case the strict solution of the equation (2) is a complex problem, but it is possible to point out at least three situations where there is a relatively simple solution:

1) Let the output signals of the receivers be stationary and stationarily bound processes and the length of the realization T be appreciably greater than the standard correlation radius of the signal. The solution of this problem was obtained in [6];

2) Let the one-dimensional signal be observed, that is, N=1; then the correlation function and the solution are represented by the series:

$$K(t,s) = \sum_{n=1}^{\infty} \lambda_n \varphi_n(t)\varphi_n(s), \quad h(t,s) = \sum_{n=1}^{\infty} \frac{\lambda_n}{\lambda_n + N_0}\varphi_n(t)\varphi_n(s), \quad (16)$$

containing the eigenvalue λ_n and the eigenfunction $\phi_n(t)$ of the integral equation:

$$\lambda_n \varphi_n(t) = \int_0^T K(t,s) \varphi_n(s) ds \; ; \qquad (17)$$

3) If the spectral signal/noise ratio is small, from (5) it follows that

$$h(t,s) = \frac{K(t,s)}{N_0} + O\left(\frac{\lambda_n}{N_0}\right), \quad \text{for} \quad \frac{\lambda_n}{N_0} \to 0, \qquad (18)$$

and, consequently, the covariation matrix can be taken as the approximate solution.

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PROBLEM OF SELECTING THE OPTIMAL PARAMETERS OF AN INTEGRATOR WHEN PROCESSING WIDE-BAND SIGNALS RECEIVED ON A SCANNING ANTENNA

By M. A. Antonets, B. M. Salin, pp 89-91

Let us consider the problems of processing wide-band noise signals received on an antenna that scans with no angular velocity. The radiation pattern of the linear antenna is related to the sensitivity distribution of the individual elements A(r) with respect to the antenna aperture by the Fourier transformation [1]:

$$\Gamma(\psi) = \frac{1}{D}\int_{-D/2}^{-D/2} A(z)\, e^{iz\psi}\, dz, \qquad (1)$$

where D is the dimension of the linear antenna. The signal from the output of the linear antenna V_c arrival of a plane wave $F_c(t)$ at the antenna with direction ϕ_0 with respect to the normal can be expressed in the form:

$$V_c(t,\varphi_0) = \frac{1}{D}\int_{D/2}^{-D/2} A(z)\, F_c\left(t - \frac{z\sin\varphi_0}{c}\right) dz, \qquad (2)$$

where C is the propagation rate of a flat antenna. From expression (2) when considering (1) by the Fourier transformation it is possible to obtain the dependence of the output signal spectrum of the antenna on the spectrum of the incoming radiation:

$$G^{(1)}_{\text{вых.с}}(\omega,\varphi_0) = G^{(2)}_{\text{вх.с}}(\omega)\,\Gamma\left(\frac{\omega\sin\varphi_0}{c}\right). \qquad (3)$$

Key: 1. out; 2. inp

Let us assume that the antenna is rotated around a central element with an angular velocity v, ϕ=vt. For convenience let us take the time of passage of the central beam of the antenna through the target as the origin of the time, that is, ϕ_c=0 for t=0. Then for certain restrictions on the analysis time T and the scanning rate v the expression (3) turns out to be correct for sliding spectra of the output and input signals:

$$G^{(1)}_{\text{вых } c}(\omega,t_n) = G^{(2)}_{\text{вх } c}(\omega,t_n)\,\Gamma\!\left(\frac{\omega\sin\varphi_n}{c}\right). \tag{4}$$

Key: 1. out; 2. inp

The window of analysis T is selected beginning with the inequality:

$$(\mathcal{D}\sin\varphi_n/c) \ll T < (c/\mathcal{D}\,v\,f_{\max}). \tag{5}$$

Let us consider the signal (the plane wave arriving at the antenna from the direction $\phi=0$) a stationary random process with spectral power density $S_c(\omega)$, and the noise a stationary process with spectral power density which depends on the direction of orientation of the antenna in space. Thus, the square of the modulus of the sliding spectrum of the antenna exit is the sum of the signal and the noise component:

$$|G_{\text{вых}}^{(1)}(\omega,t_n)|^2 = |G_{\text{вх.c}}^{(2)}(\omega,t_n)|^2\cdot|\Gamma(\frac{\omega\sin\varphi_n}{c})|^2 + |G_{\text{вых ш}}^{(3)}(\omega,t_n)|^2. \tag{6}$$

Key: 1. out; 2. inp signal; 3. out noise

In order to isolate the signal component let us carry out correlation processing of the spectra (6) with some function $K(\omega, t_n,\phi)$:

$$R(\varphi) = \int_{\omega_1}^{\omega_2}\sum_{n=1}^{N}|G_{\text{вых}}(\omega,t_n)|^2\cdot K(\omega,t_n,\varphi)\,d\omega. \tag{7}$$

Key: 1. out

The ratio of the constant component $R(\phi)$ caused by the signal part $G_{\text{out signal}}$ to the dispersion $R(\phi)$ for $\phi=0$ defines the signal/noise ratio after correlation processing:

$$Q^{(1)\,\text{сигнал}}_{(2)\,\text{шум}} = \left(\sum_n^N\sum_m^M\langle|G^{(3)}_{\text{вых c}}(\omega_m,t_n)|^2\rangle K(\omega_m,t_n)\right)\Big/\left(\sqrt{\sum_n^N\sum_m^M\sigma^2_{ш}(\omega_m,t_n)K^2(\omega_m,t_n)}\right) \tag{8}$$

Key: 1. signal; 2. noise; 3. out signal

Here the integral with respect to ω is replaced by the sum; the breakdown interval with respect to ω is selected equal to $2\pi/T$ for independence of the reckoning points. The dispersion of the sum (7) therefore is found as the sum of the dispersions $\sigma^2_{\text{noise}}(\omega_m, t_n)$ at individual points (hereafter we shall set $\sigma^2_{\text{noise}}(\omega_m, t_n)=S_{\text{noise}}(\omega_m, t_n)$. It is possible to show that (8) reaches a maximum for

$$K(\omega_m,t_n) = (|\Gamma(\omega_n\sin(\dot{v}\,t_n)|^2 S_c(\omega_m))/S^2_{ш}(\omega_m,t_n). \tag{9}$$

It is also possible to demonstrate that for an arbitrary angle of arrival of the signal at the antenna the expression $K(\omega_m, t_n, \phi)$ has the form:

$$K(\omega_m, t_n, \varphi) = (|\Gamma(\omega_m \sin(vt_n - \varphi))|^2 S_c(\omega_m, \varphi)) / S_w^2(\omega_m, t_n)$$

and the signal/noise ratio for $R(\phi)$ will have the maximum at ϕ coinciding with the angle of arrival of the radiation at the antenna.

Thus, the above-discussed processing procedure requires for its realization that the matrix of the sliding spectrum of the output signal be obtained, the noise spectra be measured and the operation of convolution of the spectra of the output signal and the noise with a two-dimensional radiation pattern of the antenna be performed. The enumerated operations, although realizable, are quite awkward; therefore the question reasonably arises of the gain which can be obtained using the given procedure by comparison with the ordinary filtration, detection and averaging identical for all frequencies. For this purpose we shall find K in the form $\hat{K}(\omega, t_n) = \eta(\omega)\xi(t_n)$. This factor is easily realized in the form of a filter with frequency characteristic equal to $\sqrt{\eta(\omega)}$, the detector and the integrating element. It is quite difficult to find the form of R; therefore let us simplify the problem and assume that the spectral density of the noise power at the antenna output does not depend on the orientation of the antenna

$$S_w(\omega, t_n) = S(\omega). \qquad (10)$$

On satisfaction of (10) R can be found, investigating the integral equation (12) at the extremum.

$$\frac{\hat{Q}}{Q} = \frac{\sum_m^M \sum_n^N \tilde{\Gamma}^2(\omega_m, t_n)\tilde{\eta}(\omega_m)\xi(t_n)}{(\sum_m^M \sum_n^N \tilde{\Gamma}^4(\omega_m, t_n))^{1/2} (\sum_m^M \tilde{\eta}^2(\omega_m))^{1/2} (\sum_n^N \xi^2(t_n))^{1/2}} \qquad (11)$$

Here: Q is the ratio of the signal to the noise when using K, $\tilde{\eta} = S_{noise} \cdot \eta$, $\tilde{\Gamma}^2 = |\Gamma|^2 S_{signal}/S_{noise}$. Expression (11) is always less than 1 and indicates how the processing efficiency differs with K and $\hat{K}$. The problem of finding the maximum (11) and also the form of $\tilde{\eta}(\omega_m)$ and $\xi(t_n)$, for which this maximum is reached is solved as follows. The eigenfunctions and eigenvalues of the operators $\tilde{\Gamma}^{2*}\tilde{\Gamma}^2$ and $\tilde{\Gamma}^2\tilde{\Gamma}^{2*}$ defined by the expressions (12) and (13) for $Q = \{Q_m\}$ and $Q = \{Q_n\}$ are found:

$$(\tilde{\Gamma}^{2*}\tilde{\Gamma}^2 Q_n)_i = \sum_m \tilde{\Gamma}^2(\omega_m, t_i) \sum_n \tilde{\Gamma}^2(\omega_m, t_n) Q(t_n); \qquad (12)$$

$$(\tilde{\Gamma}^2\tilde{\Gamma}^{2*} Q_m)_j = \sum_n \tilde{\Gamma}^2(\omega_j, t_n) \sum_m \tilde{\Gamma}^2(\omega_m, t_n) Q(\omega_m). \qquad (13)$$

It is possible to show that the maximum of expression (11) is reached for $\tilde{\eta}(\omega)$ equal to the first eigenfunction $Q^1(\omega_m)$ of the operator $\tilde{\Gamma}^2\tilde{\Gamma}^{2*}$, and $Q^1(t_n)$ equal to the first eigenfunction of the operator $\tilde{\Gamma}^{2*}\tilde{\Gamma}^2$. The magnitude of the maximum (11) is determined by the first eigenvalue (14) μ_1^2 of the operators (12), (13)

$$\hat{Q}/Q = \mu_1 / (\sum_n \sum_m \tilde{\Gamma}^4)^{1/2}. \qquad (14)$$

By the calculated parameter $\hat{Q}/Q$ for the specific type of antenna and the specific type of noise spectra it is possible to make a decision regarding the choice of the specific processing procedure. For $\hat{Q}/Q=1$ it is possible to use a simpler procedure with the use of a filter and detection; for $\hat{Q}/Q<1$ it is necessary to use spectral analysis and the convolution operation.

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MAXIMUM LIKELIHOOD PROCESSING OF BAND SIGNALS

By V. I. Chaykovskiy, pp 91-94

In accordance with the theory of optimal signal detection lasting T with unknown parameters (delay time τ, doppler frequency ν and random initial phase in amplitude) three basic versions of the signal processing algorithms against a background of stationary gaussian noise are possible.

One of the versions is a system with line scanning of the likelihood function along the frequency axis. The functioning algorithm of such a system is:

$$\ell(\beta, Y) = A\left|\int_T Y(t) G^*(t-\tau) e^{-j\nu t} dt\right| = A\left|\int_T Y(t+\tau) G^*(t) e^{-j\nu t} dt\right|, \tag{1}$$

where $Y=\gamma S_\beta+X$ is the vector envelope of the complex signal, X is the vector of the complex envelope of the noise, S_β is the information parameter vector of the signal, γ is its complex amplitude. A $\exp[j\phi]$; G is the envelope of the reference signal. In the discrete representation this expression has the form:

$$\begin{aligned}\ell(\beta, Y) &= A\left|\sum_{k=0}^{N-1} Y(k\Delta\tau) G^*[(k-n)\Delta\tau] \exp[-jp\Delta\nu k\Delta\tau]\right| = \\ &= A\left|\sum_{k=0}^{N-1} Y[(k+n)\Delta\tau] G^*(k\Delta\tau) \exp[-jp\Delta\nu k\Delta\tau]\right|,\end{aligned} \tag{2}$$

where k, p are numbers from the natural series.

As follows from the natural series of presented expressions, during processing it is necessary in each step of the shift with respect to time $\Delta\tau$ to fix the correlation product of the complex observation envelopes and the reference signal and realize its Fourier transformation with respect to the entire set N of reckoning for each of the possible values of the frequency bias $p\Delta\nu$. The operation of the system in real time requires recirculation of the input information and matching of the type of operation of the Fourier transformation processor to the rate of formation and input of the reckonings of the correlation product. After the formation of the modulus values of the Fourier coefficients, a decision is made regarding the presence of a useful signal, and the parameters are estimated with respect to the intensity of the global maximum of the sufficient statistics of the likelihood function and its position in the coordinate field $k\Delta\tau$ and $p\Delta\nu$.

Another version of the processing algorithm can be represented in the form of line scanning of the likelihood function along the time axis. The algorithm for the functioning of such a system is obtained from (1) on the basis of the Parseval theorem:

$$\ell(\beta, Y) = \frac{A}{2\pi}\left|\int_{-\infty}^{\infty} Y_1(\omega) G_1^*(\omega - \nu) e^{j\omega\tau} d\omega\right| = \frac{A}{2\pi}\left|\int_{-\infty}^{\infty} Y_1(\omega + \nu) G_1^*(\omega) e^{j\omega\tau} d\omega\right|. \qquad (3)$$

Using expression (3) and the basic principles of the theory of the discrete Fourier transformation, it is possible to obtain the discrete version of this algorithm:

$$\begin{aligned} \ell(\beta, Y) &= \frac{A}{M}\left|\sum_{p=0}^{M-1} Y_1(p\Delta\nu) G_1^*[(p-n)\Delta\nu] \exp[jp\Delta\nu k\Delta\tau]\right| = \\ &= \frac{A}{M}\left|\sum_{p=0}^{M-1} Y_1[(p+n)\Delta\nu] G_1^*(p\Delta\nu) \exp[jp\Delta\nu k\Delta\tau]\right|. \end{aligned} \qquad (4)$$

From the presented expressions it follows that in the first processing step, the Fourier transformation Y_1 is determined for the complex envelope $Y(t)$ of the input signal given in the entire interval of possible delays τ essentially exceeding the duration of the reference signal. The results of the transformation are subjected to compression and recirculation in time. The recirculating sequence of spectral reckonings in each recirculation cycle is multiplied times the series of spectral reckonings of the reference signal correspondingly shifted with respect to frequency, and it forms the correlation products in the spectral region. Each such correlation product is subjected to the inverse Fourier transformation with respect to the entire set M of possible delays τ. When operating in real time the rate of the conversion must be matched with the rate of formation and input of the correlation product. As a result of this processing, the complete surface of the sufficient statistics is formed as a set of cross sections parallel to the delay axis. The global maximum of this surface, just as in the preceding case, is used for decision making and estimating the parameters of the useful signal. It must be noted that usually a sectional processing of the continuously developing input sequence in mutually intersecting intervals is used [1]. Another optimal processing algorithm with scanning along the frequency axis can be obtained from expression (3) if we consider that the product $Y_1(\omega)e^{j\omega t}$ can be considered as a Fourier transformation coinciding by definition with the matched instantaneous or sliding spectrum of the complex envelope of the input signal in the interval T [2]:

$$\ell(\beta, Y) = \frac{A}{2\pi}\left|\int_{-\infty}^{\infty} Y_{1\tau}(\omega) G_1^*(\omega - \nu) d\omega\right| = \frac{A}{2\pi}\left|\int_{-\infty}^{\infty} Y_{1\tau}(\omega + \nu) G_1^*(\omega) d\omega\right|. \qquad (5)$$

In the discrete representation:

$$\ell(\beta,\gamma)=\frac{A}{N}\left|\sum_{p=0}^{N-1}Y_{1k}(p\Delta\nu)G_1^*[(p-n)\Delta\nu]\right|=\frac{A}{N}\left|\sum_{p=0}^{N-1}Y_{1k}[(p+n)\Delta\nu]G_1^*(p\Delta\nu)\right|. \quad (6)$$

Thus, in the first phase of the processing for each value of the delay parameter $k\Delta\tau$ in real time the sliding spectrum $Y_{1\tau}$ of the complex envelope of the input signal is defined. The interval of analysis of the spectrum T in this case is matched with the size of the reference signal. The realizations of the sliding spectrum are subjected to compression and recirculation and in each recirculating cycle multiplied with the realizations of the spectrum of the complex envelope of the signal $G_1''(\omega-\nu)$ shifted correspondingly with respect to frequency. The terms of the correlation products obtained in this way are summed, forming in each step $\Delta\tau$ cross sections of the sufficient statistics of the likelihood function parallel to the frequency axis. Inasmuch as determination of the sliding spectrum in the first processing level is possible on the basis of the well-known recurrent procedure which is highly economical with respect to computation expenditures [3], and in the second step the processing reduces to smoothing the sliding spectrum with weight determined by the spectrum of the reference signal, then the application of the likelihood method of processing based on the sliding analysis of the spectrum can turn out to be highly effective, especially for small values of the reference signal base. The effectiveness of the digital processing system is characterized by the volume of computational expenditures or the number of most tedious operations of complex multiplication expended in the formation of one cross section, for example, along the frequency axis. On comparison of the above-investigated system with respect to the total computational expenditures it is expedient to introduce the concept of complexity of the processing N equal to the product of the duration of the reference signal T_G times the width of the operating frequency band ΔF, $N=T_G(\Delta F_c+\Delta F_D)$, where ΔF_c is the spectral width of the useful signal, ΔF_D is the interval of possible frequency shifts. If in this case the digitalization interval with respect to time $\Delta\tau$ and with respect to frequency $\Delta\nu$ is selected in accordance with the Kotel'nikov theorem, then the size of the cross section of the sufficient statistics along the frequency axis required during the processing will be N. The size of the filtering sequence M can also be expressed in terms of the amount of complexity in the processing. For a system with frequency scanning it is always greater than N and in the worst case it is equal to 2N. For systems with scanning with respect to time and sectioning of the processing it also is taken equal to 2N. Finally, the size of the filtering sequence (the number of readings of the spectrum of the complex envelope of the reference signal taken into account) when processing the spectrum on the basis of a sliding analysis coincides with the size of the base of the reference signal Q<<N. Considering what has been discussed above, the volume of the computational expenditures consumed for the formation of one surface cross section of the sufficient statistics is easily obtained as a result of analyzing the algorithms for the functioning of the above-investigated likelihood processing systems (2), (4) and (6). For realization of the processing on the basis of the progressive algorithms of fast

Fourier transformations and recurrent algorithms of sliding analysis, the corresponding expenditures with respect to the number of complex operations of multiplication are defined as follows: for systems with frequency scanning α_1=N(2+log$_2$ 2N); for systems with time scanning and sectioning α_2=(N+1)xlog$_2$ 2N+Q; for systems with sliding spectral analysis α_3=N(Q+1) for Q>1 and α_3=N for Q=1. The analysis of the presented expressions for different values of the complexity of the processing N and the base dimensions of the useful signal Q indicates that the comparative effectiveness of the various systems depends on the relation of the processing complexity, the base of the signal, the band of the spectrum and the maximum possible bias of its frequency. For small values of the base of the signal ($Q \leq 32$) and the realized possible bias of its frequency ($\Delta F_D/\Delta F_C \gg 1$), the system with sliding analysis of the spectrum is the most effective. For large values of the signal base ($Q \geq 128$) and significant bias of its frequency, preference must be given to the system with frequency scanning. The system with time scanning is most effective when processing a signal with a large base ($Q > 128$) and insignificant ($\Delta F_D/\Delta F_C=1$) frequency bias. The presented recommendations estimated by structural and functional complexity must be taken into account when designing the information systems for complete maximum likelihood processing for operation in the location search mode.

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DETECTION CHARACTERISTICS IN THE CASE OF INCOHERENT SOUNDING

By V. V. Karavayev, V. V. Sazonov, pp 95-97

In cases where it is necessary to perform a fast survey of the given spectrum, the use of phased arrays forming a narrow transmitting beam is not always expedient.

In [1] a procedure was proposed for constructing the radiating system in which the excitation phases of the individual radiators vary in accordance with a pseudorandom code, and reception is carried out using a set of filters, each of which is matched to the code of the signal arriving from a given angular direction. This system has a radiation pattern, the width of which does not depend on the geometric dimensions of the transmitting array, but the realization of the receiving part is very complex.

Thus, let us investigate the transmitting system made up of a quite large number of elements separated from each other which simultaneously emit signals with uncontrolled noise-like phase in the same angular sector. The signal of each transmitter on reflection from the object acquires random modulation caused by the random scattering cross section. This modulation for different transmitters will be different, for as a result of their spatial separation the object is visible on different angles of approach. The sum of the signals emitted by all of the transmitters comes to the receiver. The processing of the received signal consists in band filtration and quadratic accumulation of it during its duration T:

$$z=\int_0^T\left|\sum_{j=1}^{N}A_j m_j(t)+n(t)\right|^2 dt,$$

where A_j are the random variables describing the fluctuations of the scattering cross section of the object on irradiation of it by the j-th transmitter which emits the signal $m_j(t)$, $n(t)$ -- gaussian noise. The distribution law of the output effect z can be found by averaging the provisional (for forced A_j) distribution law $P(z/\Lambda)$ described by the law x^2 with $2m=2\Delta fT$ degrees of freedom, where Δf is the effective width of the signal spectrum, by the random dispersion of the signal component which in the case of the Rayleigh model of the reflected signal is also distributed by the law x^2 with 2n-N degrees of freedom and the dispersion ρ^2 for one degree. Here, in the case of large signal/noise ratios the integral law has the form:

$$P(z)=\int_{z}^{\infty}p(z')dz'=\frac{z^{(m+n-1)/2}}{\rho^{m+n-1}\Gamma(n)2^{n+m-2}}\sum_{\ell=0}^{m-1}\frac{2^{\ell}K_{n-m+1+\ell}(\sqrt{z/\rho^{2}})}{\Gamma(m-\ell)(z/\rho^{2})^{\ell/2}}, \quad (1)$$

where $k_{\gamma}(z)$ is the modified Hankel function. Formula (3) is valid for integral m. For semiintegral m, the analytical expression for P(z) is also possible which we shall not present here. Fig 1 shows the dependence of the threshold signal/noise ratio on $m=\Delta fT$ with a probability of false alarm of 10^{-6} and n=5 calculated by formula (3). In the same figure the horizontal lines depict the corresponding threshold levels for coherent processing. The minimum losses are determined by the distance from these lines to the extremal points of the corresponding curves. These losses are small, for example, for D=0.9 they amount to 1.4 decibels.

Another scheme for constructing the system is possible in which for the suppression of the fluctuations of the signal reflected from the object instead of the spatial separation, frequency separation is used. This type of system is made up of the group of matched incoherent transmitters, the frequency of which is tuned from sending to sending. Simultaneously, the heterodyne frequency of the receiving system is also tuned so that the transmission frequency of each pulse remains unchanged. On reception quadratic accumulation for the duration of the signal takes place, and interpulse accumulation during irradiation of the individual angular cell. The output effect of this system has the form

$$z=\sum_{j=1}^{n}z_j=\sum_{j=1}^{n}\int_0^T|A_j m_j(t)+n_j(t)|^2dt,$$

where $m_j(t)$ and $n_j(t)$ are signal and noise components of the j-th separation branch independent of each other at the output of the intermediate frequency amplifier of the receiver, A_j are random variables describing the fluctuations of the scattering cross section of the target on the j-th frequency, and is the number of frequency separation branches.

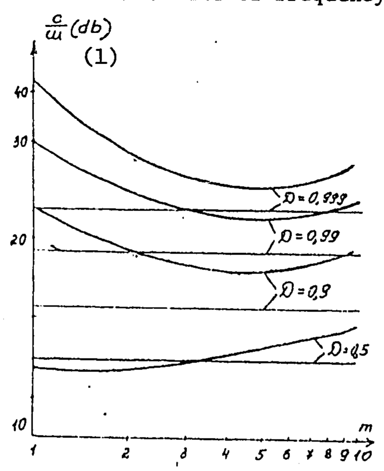

Figure 1

Key: 1. decibels

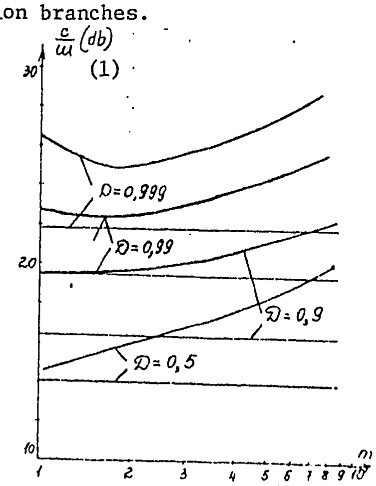

Figure 2

Key: 1. decibels

The detection characteristics for this system are most simply considered by the method of mathematical simulation. They are presented in Fig 2. From the graphs it is obvious that in the given case the losses are somewhat greater than in the system with spatial separation of the transmitter.

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EFFECT OF CONSTANT ACCELERATION ON WIDE-BAND NOISE SIGNALS DURING MUTUAL CORRELATION PROCESSING

By K. I. Malyshev, pp 97-98

Let us consider the influence of the constant acceleration on the form of the mutual time-frequency correlation and let us discuss in more detail the dependence of the magnitude of the correlation on the acceleration. Taking, just as in [1], the signal in the form of a long segment of stationary noise, after statistical averaging we obtain the mutual time-frequency correlation coefficient in the following form:

$$R(v,\tau)=\int_{-1/2}^{1/2} R_n[\tau+(\beta_v u/f_0)-(\beta_a u^2/f_0)]du, \qquad (1)$$

where R_n is the noise autocorrelation coefficient, τ is the time delay, $\beta_v = v f_0 T/c$, $\beta_a = a f_0 T^2/2c$, v is the mismatch with respect to the velocity between the received and reference signals, a is the acceleration, c is the speed of sound, T is the signal duration, f_0 is the carrier frequency; the time origin is at the middle of the received signals ans the effect of the movement on the integration limits is negligible. In contrast to [1] a study is made here of the one-way signal propagation.

Introducing the representation $R_n(\tau)=R_0(\tau)\exp(j2\pi f_0\tau)$ and estimating the integral in (1) by the stationary phase method, which is acceptable for not very wide-band signals for the correlation coefficient modulus, we obtain:

$$|R(v,\tau)|=|R_0[\tau+(\beta_v^2/4f_0\beta_a)]|\cdot\{[C(x_1)+C(x_2)]^2+[S(x_1)+S(x_2)]^2\}^{1/2}/(4|\beta_a|)^{1/2}, \qquad (2)$$

where C and S are the Fresnel integrals, $x_{1,2}=|\beta_a|^{1/2}\pm(\beta_v/|\beta_a|^{1/2})$. It is possible to show that the maximum $|R|$ is defined by the expression:

$$|R|_{max}=\{[C^2(\sqrt{|\beta_a|})+S^2(\sqrt{|\beta_a|})]/|\beta_a|\}^{1/2}. \qquad (3)$$

From (3) it follows that $|R|_{max}$ is determined by one parameter β_a. The value of a for which $|R|_{max}=\varepsilon$ will be found from the expression $|a|=2\alpha C/(f_0T^2)$ where $\alpha=\alpha(\varepsilon)$ is determined from (3). With respect to form and obtained values of the acceleration this expression agrees with the results of references [1, 2]. The time cross sections of the different correlation coefficient modulus, according to (2) do not depend on the acceleration. The frequency cross sections depend both on the acceleration and on the signal band but they do not depend on its duration. For sufficiently large β_a the width of the frequency cross section is $\Delta\beta_v \sim 4(|\beta_a|/\xi)^{1/2}$ where $\xi=\Delta f/f_0$, Δf is the signal band. In order to estimate the limits of applicability of the results obtained, the calculation was made on a digital computer of the time ($\beta_v=0$) and frequency ($\tau=0$) cross sections of the modulus of the mutual time-frequency correlation coefficient for the signal with constant spectral density in the band Δf. The calculation demonstrated that $|R|_{max}$ depends weakly on the signal band on variation of ξ from 0.2 to 1.2 and with an error not exceeding 10% it is determined by the expression (3). The form of the time cross sections and their width on the 0.5 level with respect to the maximum correlation depends weakly both on the magnitude of the acceleration and on the signal band. For $|\beta_a|<1$ $\Delta\beta_v$ on the relative level of 0.5 it depends weakly both on the acceleration and on the signal band and, when $|\beta_a|>1$ it increases with an increase in β_a and a decrease in ξ. The calculated values of $\Delta\beta_v$ are presented in Fig 1. They are noticeably less than the estimates by formula (2). Thus, from the calculations it follows that formula (2) can be used only for a rough estimate of the frequency width of the cross sections for $|\beta_a|>1$. The calculations also show that formula (2) qualitatively correctly transmits the asymmetry of the modulus of the mutual correlation coefficient.

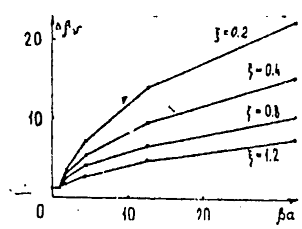

Figure 1. Dependence of $\Delta\beta_v$ on β_a

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NOISEPROOFNESS OF MATCHED FILTRATION OF WIDE-BAND SIGNALS WITH FREQUENCY DISTORTIONS OF THE ECHO AND INTERFERENCE SPECTRA

By A. A. Misnik, K. V. Filatov, pp 98-100

A study is made of the case of processing the mixture of distorted echo and reverberation and noise interference having essentially nonuniform spectral densities in a matched filter. The losses are determined with respect to the case of receiving an undistorted signal against a background of white noise. Let us assume that the signal is propagated in a single-beam channel, including an aqueous medium and the receiving and transmitting antenna, phase distortions are absent, the equivalent radius of the circuit is quite small and does not depend on the frequency. Normalizing the moduli of the spectra of the emitted signal $S_t(f)$ and the echo $S(f)$ and also the spectral densities of the noise and reverberation interference $N(f)$ and $R(f)$ to their values on the central frequency f_0 of the signal spectrum and introducing the parameter $B=N(f_0)/R(f_0)$, we write the signal/noise ratio at the output of the matched filter [1]:

$$Q^2 = \frac{U_H^2}{P_{n\,\text{вых}}}_{(1)} = \frac{\left[1/\pi\, S(f_0)\int_0^\infty S_H(f) S_{tH}(f)\,df\right]^2}{1/\pi\, R(f_0)\int_0^\infty [B N_H(f) + R_H(f)] S_{tH}^2(f)\,df},$$

Key: 1. interference at the output

where $U_H^2, P_{n\,\text{вых}}$ is the maximum value of the response of the matched filter and the interference power at its output. Let us denote by Q_0 the signal/noise ratio at the output of the matched filter in the case of reception of an undistorted signal against a background of white noise with equivalent spectral density $G_э = R(f_0)(B+1)$ and let us introduce the loss coefficient

$$\eta^2 = \frac{Q_0^2}{Q^2} = \frac{\int_0^\infty S_{tH}^2(f)\,df \cdot \int_0^\infty [B N_H(f) + R_H(f)] S_{tH}^2(f)\,df}{\left[\int_0^\infty S_H(f) S_{tH}(f)\,df\right]^2 (B+1)}. \qquad (I)$$

The finite expression for η^2 will be written for sounding signals with spectra $S_{tH}(f) = \text{rect}[(f-f_0)/F]$ which are planar within the limits of the frequency band F under the assumption that for emission and reception one and the same antenna is used in the form of a plane phased array and that the spectral density of the noise interference is inversely proportional to the square of the frequency. Using the known expressions for the intensities of the echoes and the interference at the output of the receiving antenna [2] and assuming that $\xi=F/f_0\leq 0.2$, the normalized spectra can be represented in the form:

$$S_H^2(f)=\left(\frac{f}{f_0}\right)^2 \exp[\alpha(f-f_0)],\ R_H(f)=\left(\frac{f}{f_0}\right)\exp[\alpha(f-f_0)],\ N_H(f)=\left(\frac{f_0}{f}\right)^4, \quad (2)$$

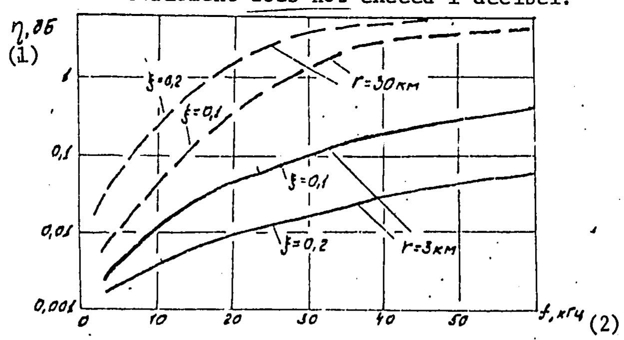
where $\alpha=-2.49\cdot 10^{-2}r\sqrt{f_0}$, r is the distance from the object of location, km, f_0 is in kilohertz. Substituting (2) in (1) and calculating the integrals, we obtain:

$$\eta^2=\frac{F^2\{b[1+\xi^2 5/6]-(A^2/\alpha F)[1+\xi/2+1/\alpha f_0-(1-\xi/2+1/\alpha f_0)/A^4]\}^2}{\{-(2A/\alpha)[1+\xi/2+2/\alpha f_0-(1-\xi/2+2/\alpha f_0)/A^2]\}^2(b+1)}, \quad (3)$$

where A=exp(-aF/4).

Let us note that although the parameter b is a function of the variables r and f_0, it cannot be excluded from (3) as an independent variable, for it depends on the scattering coefficient α_{Rh} and the intensity of the noise in the aqueous environment which are not functionally related to r and f_0.

In the figure we have the functions $\eta^2(f_0, r, \xi)$ calculated according to (3) for B=1. Analysis of the function obtained indicates that in the region of the sense of values r, f_0, ξ which are usable in practice the magnitude of the loss coefficient does not exceed 1 decibel.



Key: 1. decibels; 2. kilohertz

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SIMULATION OF OPTIMAL MUTUAL CORRELATION PROCESSING OF SIGNALS

By. K. P. L'vov, pp 100-102

Assuming that the emitted signal is a complex signal and using the known assumption about the autocovariation reverberation function, the reference signal of the correlator in the frequency range can be written as follows [1], [2]:

$$H(\omega)=g_{zc}(\omega)/(G_n+\frac{I_p}{E_c}|g_{z_c}(\omega)|^2), \tag{1}$$

where $g_{zc}(\omega)$ is the Fourier transformation of the complex envelope $Z_c(t)$ of the emitted signal, G_n is the spectral density of the "white" noise power, I_p is the mean intensity of the reverberation, $E_c = \int_0^{T_c}|Z_c(t)|^2dt$. In the presence of a signal $AZ_c(t-\tau)$ at the input of the system, the output signal is represented in the form

$$\underset{(1)}{z_{\text{вых}}}(t)=\frac{A}{2\pi}\int_{-\pi\omega}^{\pi\omega}g_{z_c}(\omega)H^*(\omega)\exp[j\omega(t-\tau)]d\omega. \tag{2}$$

Key: 1. out

The signal/noise ratio at the channel output is [2];

$$q^2=\frac{A^2}{2\pi}\int_{-\pi\omega}^{\pi\omega}[|g_{z_c}(\omega)|^2/(G_n+\frac{I_p}{E_c}|g_{z_c}(\omega)|^2)]d\omega. \tag{3}$$

Let us consider the possible structure of the discrete analog of the optimal mutual correlation processing (1). Let us place the lattice function $Z_c[kT]$ in correspondence to the continuous function $Z_c(t)$, where T is the digitalization interval, k = 0, 1, 2, ..., N-1; N-1 = entier (T_c/T). The discrete Fourier transformation (DFT) for $Z_c[kT]$:

$$g_{z_c}(\omega T)=\sum_{k=0}^{N-1}z_c[kT]\exp(-j\omega T_k). \tag{4}$$

In the frequency range let us define the reference signal of the discrete optimal mutual correlation processing as:

$$H(\omega T) = g_{z_c}(\omega T)/(G_{nT} + \frac{I_p T}{E_c}|g_{z_c}(\omega T)|^2), \tag{5}$$

where G_{nT} is the spectral density of the power of the complex envelope of the discrete "white" noise. If only $AZ_c[(k-m)T]$ is present at the output of the discrete system and assuming that $\tau = mT$, then considering (5), the output signal will be defined using the inverse DFT as:

$$\tilde{z}_{\text{вых}}[kT] = \frac{A}{2\pi}\int_{-\pi}^{\pi} g_{z_c}(\omega T) H^*(\omega T) \exp[j\omega T(k-m)]\, d\omega T. \tag{6}$$

(1)

Key: 1. out

Considering (5) and (6) the output ratio (signal/noise):

$$\tilde{q}^2 = \frac{A^2}{2\pi}\int_{-\pi}^{\pi} [|g_{z_c}(\omega T)|^2/(G_{nT} + \frac{I_p T}{E_c}|g_{z_c}(\omega T)|^2)]\, d\omega T. \tag{7}$$

After transformations we have:

$$\tilde{q}^2 \simeq \frac{A^2}{2\pi}\int_{-\pi/T}^{\pi/T} [|g_{z_c}(\omega)|^2/(G_{nT} + \frac{I_p}{E_c}|g_{z_c}(\omega)|^2)]\, d\omega. \tag{8}$$

From a comparison of (8) and (3) it follows that if the digitalization interval satisfies the condition $T \leq 1/\omega$, the discrete processing is equivalent to analog. Let us consider (4) for the DFT of the lattice function $Z_c(kT)$. The values of (4) at the points that are multiples of π/T_N are equal to:

$$g_{z_c}(\frac{\pi}{N}m) = \sum_{k=0}^{N-1} z_c[kT] \exp(-j\frac{\pi}{N}km). \tag{9}$$

Using (6), the values of (5) at the points $\pi m/N$:

$$H(\frac{\pi}{N}m) = [g_{z_c}(\frac{\pi}{N}m)/(G_{nT} + \frac{I_p T}{E_c}|g_{z_c}(\frac{\pi}{N}m)|^2)].$$

Figure 1 shows the result of the simulation using the procedure of [3]. The mathematical model of $Z_c(t)$ was the signal with rectangular envelope and hyperbolic frequency modulation. The frequency deviation ω = 100 hertz, T_c = 1 second, T = 1/512 sec and N = 512. Figure 1 shows the normalization of the envelopes of the signals for the case of processing only a completely known signal. Here the reference signal corresponded to various ratios of the reverberation intensities to the noise [1] $q^2_{FN} = I_p T_c/G_n$. The output signals are plotted in the figures only within the limits of $\pm 0.5\, T_c$ with respect to the maximum value. The distance between the time marks is 1/51.2 sec. The simulation time on the BESM-4 digital computer of one response is approximately 15 seconds.

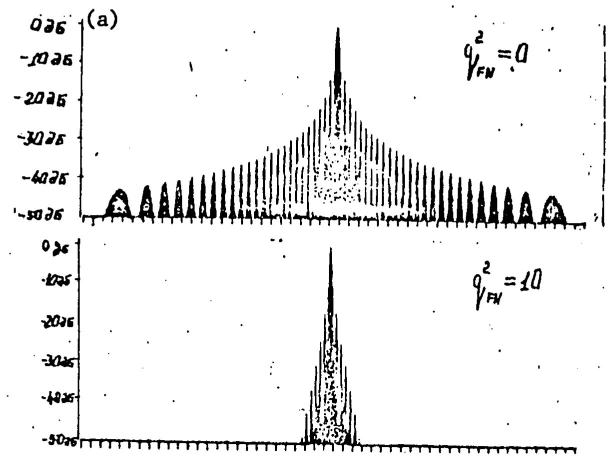

Figure 1. Envelope of the output signals for various values of q^2_{FN}.

Key: a. decibels

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ESTIMATING THE PROBABILITY OF ERRORS IN CLASSIFYING HYDROACOUSTIC SIGNALS

By Ye. L. Pasechnaya, pp 102-104

Let us consider the problem of classifying hydroacoustic signals from two sound sources, one of which is located at the surface of the ocean, and the other at depth. The ranges of possible depths of submersion of the sources do not intersect. The solution to the problem of classification is based on using the peculiarities of the distribution of the sound energy of the signals from both sound sources in the vertical plane. The depth of submersion of the sound receiver, the aperture, the slope of the axis of the directive characteristic are selected considering these peculiarities under various hydrologic conditions. The investigated classification problem can be formulated as a problem of checking two statistical hypotheses:

γ_1 is the input effect $u(t, \vec{r})$ which is caused by the noise source I at the surface;

γ_2 is the input effect $u(t, \vec{r})$ caused by the sound source II located at depth.

The solution to the classification problem in this statement leads to an optimal processing algorithm -- the likelihood ratio [1]. For the spatial processing of the signals arriving from the two sound sources, the radiation patterns in the two systems must be oriented in the direction of the arrival of the beams from the sources, and the effective width must correspond to the range of variation of the sliding angle of the beams coming from the two sources. The realization of the indicated optimal processing presents significant technical difficulties. Accordingly, it is of interest to use quasioptimal processing systems for the classification which, maintaining the basic features of the optimal algorithm, will be simple to realize. One of the versions of the construction of such a system is the two-channel receiving system containing a system with narrow and wide radiation patterns. The solution of the presence of sound sources I and II is obtained by the results of comparing the difference of the output effects of the two systems with some threshold: $V_w - V_y > \Delta$ -- sound source I; $V_w - V_y \leq \Delta$ -- sound source II. Let us define the probability of the errors and the classification, assuming that the two receiving signals are standard signal detection channels. If the product of the passband of the preselector Δf_e times the averaging time T of the low frequency filter $\Delta f_e T >> 1$, then the voltage at the output of the subtraction system is distributed in accordance with a gaussian law. In this case the total classification error is described by the expressions:

$$p = \frac{1}{2}[1 - \Phi(\frac{m_I}{\sigma_I} - \frac{\Delta}{\sigma_I}) + \Phi(\frac{m_{II}}{\sigma_{II}} - \frac{\Delta}{\sigma_{II}}), \quad \Phi(x) = \int_{-\infty}^{x} e^{-t^2/2} dt,$$

m_I, m_{II}, σ_I^2, σ_{II}^2 are the mathematical expectation and disperseness of the difference in the responses of the two systems respectively for signals from sources I or II.

The calculations of the error probability in the classification were made for standard hydrologic conditions characterized by the presence of a thermal underwater sound channel with depth of occurrence of the axis of 100 meters [summer] and the conditions of the positive refraction of the sound beams [winter]. In Figure 1 the relations are presented for the classification error probability as a function of the signal/noise ratio q_0^2 for the first source with optimal threshold minimizing the value of p under the conditions of an underwater sound channel. The apertures of the directivity characteristics in the receiving system [two pages missing from the source text].

EXAMPLE OF THE GAME APPROACH TO THE PROBLEM OF OPTIMIZING THE PARAMETERS OF A HYDROACOUSTIC INFORMATION SYSTEM

By V. V. Beresnov, A. S. Moskalenko, pp 104-107

[Two pages missing from the original source text]...

where ΔF_n is the preselector frequency band; F_m is the probability of false alarm at the "point"; J_e is the intensity of the echo; J_{loc} is the intensity of the local interference component; J_{u3} is the signal duration.

Let us transform (4) considering the restrictions assumed when deriving expression [3] and the following assumptions: absence of reverberation restrictions, the probability of false alarm in the inspection cycle of the body of water 0.1; sensitivity in the direction of the local interference -- 10%. As a result, we obtain the equality:

$$29\sqrt{\frac{40f\tau}{N_f}\left|\lg\frac{3.8\cdot 10^{-3}}{Df^2 z}\right|} = \frac{37.2\cdot 10^{-9} R_e^2 A^2 \tau}{f^2\cdot z^4\, 10^{0.0022 f^2 z}}\left(\frac{1.8\cdot 10^{-12} P^2_{\text{(a)}}}{f^2} + \frac{4.5\cdot 10^{-10} P_{\text{(b)}}}{f^2 D}\right)^{-1} \quad (5)$$

Key: a. density b. di

where A is the signal propagation anomaly; R_e is the equivalent reflection radius; $P_{density}$ is the spectral density of the local interference component; P_{di} is the spectral density of the isotropic component of the interference. The conditions of technical results of the hydroacoustic information system and also the limited possibilities of the carrier with respect to its placement have given rise to restrictions on a number of the technical characteristics of the hydroacoustic information systems:

$$1 \leq N_f \leq 40f\tau,\quad 4 \leq f \leq 20,\quad \frac{22}{f} \leq D \leq 1,\quad 0.003 \leq \tau \leq 0.3,\quad P_c \leq 500. \quad (6)$$

On the basis of the above discussion it is possible to draw the following conclusions:

1. As the vector of the parameters of the hydroacoustic information system it is expedient to use the five-dimensional vector $\chi = (f, \tau, D, N_f, N_\varphi)$, the last two components of which are necessarily integral.

2. The set χ of admissible vectors in the parameters of the hydroacoustic information system is given by the nonlinear system of restrictions (6) in which P_c, as a function of x, is defined by the equality (3).

3. As the basis for estimating the effectiveness of the hydroacoustic information system it is possible to use the function $S = S(x, z)$ defined by the equality (2) and depending both on the vector of the parameters and on the range of the hydroacoustic information system.

4. The equation (5) indicates that the range is a single valued function not only of the system parameters but also the three-dimensional vector $y = (A, R_e, P_{di})$ of undefined factors which vary in the parallepiped:

$$Y = \{y : 1 \leq A \leq 100;\ 6 \leq R_э \leq 40;\ 0{,}002 \leq P_{nu} \leq 0{,}1\}$$

and characterize the situation in the functioning range of the hydroacoustic information system.

Thus, the problem of selecting the best version of the parameters of the hydroacoustic information system is in essence the problem of optimization under conditions of indeterminacy of the vector y. In order to eliminate this indeterminacy, the flexibly understood principle of the guaranteed result was used [2], according to which it is necessary to take the following function as the criterion of effectiveness of the parameters of the hydroacoustic information system:

$$Q(x) = \min S(x, z(x,y)), \quad y \in Y, \tag{7}$$

and the solution of the problem of maximization as the optimal vector

$$Q(x) \to \max, \quad x \in \mathfrak{X}. \tag{8}$$

In other words, the problem of selecting the best version of the system parameters can be interpreted as the problem of finding the optimal strategy $x \in X$ for the person making the decision in the game with "nature." The payoff in this game is the function $S(x, r(x, y))$, and the set of strategies of "nature" coincides with Y. The analysis of the function $S(x, r)$ indicates that $Q(x) = S(x, \min_{y \in Y} z(x,y))$. In other words, based on the properties of the equation (5) it is possible to show that independently of $x \in \mathfrak{X}$ $\min z(x,y) = z(x,y^*)$, $y \in Y$, where $y^* = (1.0), 6.0, 0.1)$. This situation reflects the obvious fact that the range of the hydroacoustic information system is minimal for the least anomaly and radius of the detected object and the greatest amount of interference. Thus, for all $x \notin X$ $G(x) = S(x, r(x, y^*))$, that is, the calculation of $G(x)$ was essentially reduced to solving the equation of hydrolocation. The random search method was taken as the basis for the algorithm for the numerical solution of the maximization problem (8). In order to increase the operating efficiency of this algorithm, the specific nature of the system of restrictions was used permitting representation in them in the form which excludes the incidence of random points outside the region x. Here the best value of the parameter N_ϕ when fixing the remaining parameters was found automatically by some analytical expressions. The existence of such expressions was confirmed by the factor that the parameter N_ϕ has no influence on the range of the hydroacoustic information system.

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PROCESSING SIGNALS BY COMBINATION RECEIVERS

By S. N. Zelenskiy, S. V. Pasechnyy, pp 107-111

Let us investigate the possible paths of decreasing the dimensions of the spatial region of observation when detecting signals against a background of interference in the uniformly layered medium. Let the pressure field $p(R, t)$, as a function of the point of space of a flat-layered wave guide D_1, $R \ni D_1$, and the time t, be the solution to the wave equation with variable speed of sound with respect to depth $C_0(e)$ $(0 < z < H)$ with volumetric sources: $\Delta F \overset{D_1}{=} f$.

Lemma I. Let the region D_1 for the equation $\Psi'' = [-k^2(z) + \xi^2]\Psi$, $\operatorname{Im}\xi = 0$, consist only of the regular points, and the function $k(z) = \omega/(C_0(z)$ be an analytical function of z. Then the Green function $G(\omega; r, R)$ of the Helmholtz equation (see formula (47.10) from page 284 [1]) exists and is an analytical function of its arguments at all points of the region D_1 with the exception of the set $z^{(1)} = z^{(2)}$ and (or) $z_\perp = R_\perp$ $(z = (z_\perp, z^{(1)}),\ R = (R_\perp, z^{(2)}))$.

Theorem 1. Let the spectra of the pressure field $p(\omega, r_s)$ normal to the surface $S = \bar{D}_1 \setminus D_1$ of the velocity components $v_n(; r_s)$ and the force component $F_n(\omega; r_s)$, $r_s \in S$, decrease faster than any power $|r_s|^{-n}$ with an increase in $|r_s|$, and let the spectra of the volumetric forces $f(\omega; z)$, $z \in D_1$ decrease faster than any power $|z|^{-n}$, $\omega \in [-\Omega, \Omega]$, $0 \le \Omega < \infty$; $n = 1, 2, \ldots$; also let $p(\omega, z_s) \equiv v_n(\omega; z_s) \equiv F_n(\omega; z_s) \equiv f(\omega; z) \equiv 0$ for $|\omega| > \Omega$. Then on satisfaction of the conditions of the proved lemma I the field $p(R, t)$ is an analytical function of the argument R.

Consequently, it is possible to establish a one-to-one correspondence between the values of $p(R, t)$ for all R from D_1 and the values of the derivatives $n = 0, 1, 2, \ldots$; $\bar{n} = (n_1, n_2, n_3)$; $n_1, n_2, n_3 = 0, 1, 2, \ldots$; $D_{\bar{n}}^{n} p(R_0, t) = \partial^n p(R_0, t)/\partial R_{01}^{n_1} \partial R_{02}^{n_2} \partial R_{03}^{n_3}$ $(n = n_1 + n_2 + n_3,\ n)$ of the field p at one point R_0 of the region D_1.

Let us investigate the characteristic functional of the pressure field p(R,t), for which we shall introduce the following definitions; characteristic functional $\theta[\nu^{(0)}, \nu^{(1)}, \ldots, \nu^{(n)}, \ldots]$ of the random time functions $\mathcal{D}^n_{\bar{n}} p(R_0, t)$, $\nu^{(n)} = (\nu_1^{(n)}, \ldots, \nu_\ell^{(n)})$, where ℓ is the number of different nth order derivatives ($\ell = 2n + 1$ == in the analyzed case of a plane layered wave guide) and the characteristic functional $P[\chi]$ of the field p(R, t). By definition we have

$$\theta[\nu^{(0)}, \ldots, \nu^{(n)}, \ldots] = \left\langle \exp\left\{ i \left[(\nu^{(0)}, p)_{R_0,T} + \sum_{i=1}^{3} (\nu_i^{(1)}, \mathcal{D}^1_{\bar{\delta}_i} p)_{R_0,T} + \ldots \right] \right\} \right\rangle, \tag{1}$$

$$\bar{\delta}_i = (\delta_{i1}, \delta_{i2}, \delta_{i3}),\ \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases},\ (\nu_j^{(n)}, \mathcal{D}^n_{\bar{n}_j} p)_{R_0,T} \overset{\mathrm{Df}}{=} \int_{-\infty}^{\infty} \nu_j^{(n)} \mathcal{D}^n_{\bar{n}_j} p(R_0, t)\,dt;$$

$$\mathcal{P}[\chi] = \langle \exp\{ i(\chi, p) \} \rangle, \tag{2}$$

where the brackets < > denote the operation of probability averaging, χ and $\nu^{(n)}$, n = 0, 1, 2, ..., are the arguments of the characteristic functionals which are functions of the spatial coordinate and/or time.

Lemma 2. Let the characteristic functionals (1), (2) exist; then on satisfaction of the conditions of theorem I for almost all realizations of the random field p(R, t), the characteristic functionals $\theta[\nu^{(0)}, \ldots, \nu^{(n)}, \ldots]$ and $P[\chi]$ are related by the one-to-one mapping where:

$$\mathcal{P}[\chi] = \theta[\mu^{(0)}, \mu^{(1)}, \ldots, \mu^{(n)}, \ldots], \tag{3}$$

where

$$\mu^{(0)} = \int_{D_1} \chi(R, t)\,d^3R;\quad \mu_i^{(1)} = \int_{D_1} (R_i - R_{0i}) \chi(R, t)\,d^3R,\ i = 1, 2, 3;$$

$$\mu_k^{(2)} = \frac{1}{2} \int_{D_1} \chi(R, t)(R_k - R_{0k})^2\,d^3R,\quad k = 1, 2, 3;$$

$$\mu_k^{(2)} = \int_{D_1} \chi(R, t)(R_i - R_{0i})(R_j - R_{0j})\,d^3R,\quad (k, i, j) = (4, 2, 1), (5, 3, 1), (6, 3, 2)$$

Let us introduce the following notation: D is a singly connected subregion of the region of the plane layered wave guide D_1, the set of points of which forms the three dimensional body, S (correspondingly, ℓ) is part of some plane (correspondingly, a straight line) with arbitrary area (length) differing from zero; $C_p(D)$, $C_{pv}(S)$, $C_{pv}(D^n_{n\perp}; \ell)$, $C_{pv}(D^n_{n_2,n_3}; R_0)$ are the optimal processing systems based on the ideal measurements respectively: a) the pressures inside D; b) the pressures and the velocity component normal to S at all points S; c) the values of the pressure in the interval ℓ, the velocity components v_1 orthogonal to ℓ and also the values of $\partial^n p(R_0, t)/\partial R^n_{0\perp}$ -- the derivatives of the field p with respect to the direction $R_{0\perp}$: orthogonal to v_1 and ℓ; d) the values of the pressures $p(R_0, t)$ at the fixed point $R_0 \in D$, the velocity components $v_1(R_0, t)$ and also the values of $\partial^n p(R_0, t)/\partial R_{02}^{n_2} \partial R_{03}^{n_3}$ --derivatives with respect to the coordinates in the plane perpendicular to v_1. The notation "$D^n_{\bar{n}}\, p(R_0, t)$-receivers" will be used to denote the receivers which measure the values of the functions $D^{n}_{\vec{n}}p(R_0, t)$ at one point from D. By the ideal measurements we mean the measurements without errors. From lemma 2 we have the following statement.

Theorem 2. The likelihood ratio of the processing system $C_p(D)$, $C_{pv}(S)$, $C_{pv}(D^{n}_{n_1}, \ell)$, $C_{pv}(D^{n}_{n_1, n_2}, R_0)$ for ideal measurements of the field parameters are equal to each other. The last statement can be reformulated as follows: independently of the number of points of space in which the ideal measurements are made, use, along with the pressure receivers, of receivers that differ qualitatively from them, permits the creation of processing systems which are not inferior to the systems based on continuous processing of the field p(R, t) inside the entire domain D_1.

Let us illustrate the effectiveness of the application of $D^{n}_{\vec{n}}p$ receivers in the example of the detection of a plane wave propagated along the z-axis in an isotropic gaussian noise pressure field p(R, t) represented in the form of a superposition of plane waves. In this case the square of the detection parameter d^2 of the system made up of a point combination receiver constructed from all $D^{n}_{\vec{n}}p$-receivers, $n \leq N < \infty$ is equal to the product of the square of the detection parameter d_p^2 of the receiving system based on one point pressure gage and the value of d_k called the quality coefficient, $d^2 = d_p^2 \cdot d_k$, where $d_k = \sum_{n,n'=0}^{N} g^{nn'}_{\vec{n}\vec{n}'} \delta_{0n_1} \delta_{0n_2}$; $\vec{n} = (n_1, n_2, n_3)$, $\vec{n}' = (n_1', n_2', n_3')$, $n = n_1 + n_2 + n_3$, $n' = n_1' + n_2' + n_3'$, $n_i, n_i', i = 1, 2, 3$ where the values of $g^{nn'}_{\vec{n}\vec{n}'}$, are elements of a matrix inverse to the correlation matrix of the processes at the outputs of the $D^{n}_{\vec{n}}p$-receivers normalized for the value of the correlation function of the process at the output of the pressure receiver. Let the combination receiver consist of $D^{n}_{\vec{n}}p$-receivers $n \leq N$, $\vec{n} = (n, 0, 0)$; then for the quality coefficient we obtain $d_k = 1$; for N = 0.1; $d_k = 9/4$, for N = 2, 3. Since the signal at the outputs of the $D^{n}_{\vec{n}}p$-receivers is absent, the growth of the square of the detection parameter d^2 takes place as a result of compensation for the noise at the output of the p-receiver. In the case where the point combination receivers made up of $D^{n}_{\vec{n}}p$-receivers for $\vec{n} = (0, 0, n)$ for the quality coefficient we obtain:

$$d_k = (1 + N)^2, \quad N = 0, 1, 2, 3, \tag{4}$$

that is, the use of receivers of this type is significantly more effective. From (4), it follows that the point combination receivers for $\vec{n} = (0. 0, n)$; n = 0, 1, ..., N is equivalent to the linear equidistant antenna made up of $(1 + N)^2$ pressure receivers located relative to each other at a distance greater than the correlation radius of the isotropic field.

Let us consider the paths of reduction of the continuous processing to discrete on the basis of the $D^n_{\vec{n}}p$-receivers with acceptable natural noise in the case of finite N (for $n \leq N \gg 1$). In this case it is possible to use only the first N-term series (4). Then the value of the field p(R, t) with given error can be defined by the finite sum of the terms of the series only in some vicinity of the point R_0 of finite radius, and for small N, small radius. On the basis of the latter it is possible to break down the region of spatial time observation D into a finite number M of subregions V_i, $\bigcup_{i=1}^{M} V_i = D \times T$, in each of which with the required accuracy the truncated series (6) gives the value of the field p(R, t). With this breakdown the continuous processing is equivalent to discrete constructed on the basis of M sets of $\{D^n_{\vec{n}}p\}^{n=N}_{n=0}$-receivers, each of which is located at an inside point of the subregion V_i, i = 1, 2, ..., M or in a small vicinity of it. For N = 1 this principle is used in [2] to replace the three-dimensional region D of continuous processing based on the p-receivers, the linear base made up of p and v_n-receivers where v_n is the oscillatory velocity component normal to the linear base. Thus, in order to replace the continuous optimal space-time processing by discrete processing, in smooth differentiable fields it is sufficient to have two types of receivers: p and $D^1_{\vec{n}}p$ receivers. With an increase in N the number of points of space in which it is necessary to make the measurements will decrease, approaching one; in the case of analyticalness of p inside D. As a rule, with the exception of cases where it is possible to use holographic (optical) principles continuous processing is actually unrealizable. The fact that it is equivalent to discrete using $D^n_{\vec{n}}p$-receivers of finite nth order can have important practical application.

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PRINCIPLES OF UNBIAS AND SIMILARITY IN THE PROBLEMS OF CLASSIFICATION AND ESTIMATION OF PARAMETERS WITH A PRIORI INDETERMINACY

By Yu. Ye. Sidorov, pp 111-118

1. In this paper a brief discussion is presented of the basic principles of the theory of synthesizing unbiased and similar decision rules; the conditions are formulated under which it is possible to obtain the optimal unbiased (similar) rules; the corresponding operations of synthesis are defined for parametric indeterminacy; the conditions are determined on satisfaction of which the decision functions for a number of the most widespread nonparametric problems have the desirable property of unbias. When writing this paper, the material of the monographs [1, Chapter 1, 4 and item 1 of Chapter 5] and [2, Chapter 2], the frequency survey article [3] and monographs [4, 7] which are again referenced for certain theoretical principles, was used to the highest degree.

2. Parametric a Priori Indeterminacy. Let the set of observations $\{X\} = \{x_1, \ldots, x_n\}$ generate the statistical structure (χ, A, P) as follows: here χ is the space of all the possible values of $\{x_1, \ldots, x_n\}$ which X can assume, that is, the n-dimensional euclidian space (the observation space); A is the sigma-field of the Borel subsets χ; P is the family of probability measures (probability distributions) dominated by some sigma-finite measure defined in the measurable space (χ, A). In the case of parametric a priori indeterminacy the problem of the synthesis of the decision selection rule can be formulated as the problem of finding the rule for checking the complex hypothesis $H: \theta \; \Omega_H$ with respect to the complex alternative[1] $K: \theta \in \Omega_k$ about a multidimensional, in the general case (and possibly vector) parameter θ of the family of probability distributions $\mathcal{P} = \{P_{\theta,\vartheta}(x), (\theta, \vartheta) \in \Omega, x \in X\}$, where Ω is the parametric space which is (just as the selection space X) a

[1] We shall limit ourselves to the single-alternative situation (the classification in two classes (detection), recognition of two patterns, and so on).

region of finite dimensional euclidian space, $\Omega_H \cup \Omega_k = \Omega$, x is the observed selection, $P_{\theta v}(x)$ is the distribution density of the observed selection with respect to the sigma-finite measure containing the checked (useful) parameter θ and the arbitrary (completely undefined or disturbing) parameter v, is in the general case multidimensional. The exact value of the parameter θ is unknown, and no assignment of its a priori distribution is proposed in contrast to the classical case of the complete a priori information. The hypotheses H and K give only the classification of θ in a defined region of space Ω, classifying it as Ω_H or Ω_k, where the boundary $\omega = \{(\theta, \vartheta): \theta = \theta_0, -\infty < \vartheta < \infty\}$ between the hypotheses H and K is the set of parametric points (θ, v) with the given value of the useful parameter θ.

The use of the principle of unbias permits us to obtain the classification rules and estimates of the parameters satisfying the following important requirements: a) the probability of error of the first type must be constant for any $\theta \in \Omega_H$; b) the probability of a correct decision must be maximal for any $\theta \in \Omega_k$; c) the decision function must not depend on the distribution parameters of the observed sample. The satisfaction of these requirements insures stability of the structure of the decision making system and its optimalness for unknown distribution parameters of the observed selection. Here the system does not need any substructure under the actual values of the a priori unknown parameters. The use of such systems with fixed structures in a number of cases is more expedient than the systems with variable structure (adaptive or self-organizing) at least as a result of simplicity of realization. The effect of invariance of the structure is frequently achieved by the dependence of the threshold function of the unbiased rules on certain statistics, that is, the presence of the "floating" threshold which depends on the observation results. The principle of unbias is formalization of the impartiality of the decision making procedure. It is known that the Bayes procedure reduces to minimizing the average risk function

$$R(\theta, \varphi) = E_\theta[L(\theta, \varphi(x))], \tag{1}$$

where $E_\theta[\cdot]$ is the averaging sign, $L(\cdot\cdot)$ is the loss function, $\phi(x)$ is the decision function which for any observed selection of x establishes with what probability it is necessary to make the decision in favor of the alternative K($0 \leq \phi(x) \leq 1$). The minimizing ϕ depends, generally speaking, on the unknown value of θ. Therefore it is highly desirable to limited to the class of "impartial" decision procedure, inside which it is possible to hope to find the procedures with uniformly least risk. One such "limited" class is the class of unbiased similar decision rules.

The decision function ϕ is called unbiased if for all θ and $d_1 \in D$

$$E_\theta[L(d_1, \varphi(x))] \geq E_\theta[L(d_0, \varphi(x))], \tag{2}$$

where the averaging of $E_\theta[\cdot]$ is carried out by the distribution containing θ, d_0 is the solution which is valid for θ, d_1 is the adopted decision, D is the space of these decisions. In other words, the decision function $\phi(x)$ is unbiased if on the average $\phi(x)$ is closer to the correct decision than to any of the incorrect decisions. Generalizing the definition of (2) for any θ and θ_1, it is possible to show that $\phi(x)$ is the unbiased rule if

$$E_\theta[L(\theta_1,\varphi(x))] \geqslant E_\theta[L(\theta,\varphi(x))]. \tag{3}$$

A special case of the general definition (3) (for appropriate selection of the loss function) is the following formulation of the unbias: the rule for checking two complex hypotheses H: $\theta \in \Omega_H$ at k: $\theta \in \Omega_k$ with respect to the family of distribution $\mathcal{P}=\{P_{\theta,\vartheta}(x),(\theta,\vartheta)\in\Omega, x\in X\}$ is called unbiased if its power function $\beta_\varphi(\theta)=E_\theta[\varphi(x)]$ satisfies the inequalities

$$\begin{aligned} \beta_\varphi(\theta) &\leqslant \alpha, \quad \text{for} \quad \theta\in\Omega_H; \\ \beta_\varphi(\theta) &\geqslant \alpha, \quad \text{for} \quad \theta\in\Omega_K. \end{aligned} \tag{4}$$

where $\alpha=\sup_{\Omega_H} P\{X\in S_1\}$ the first type error probability (the dimension of the rule $\phi(x)$ or the dimension of the critical region S_1). In other words, when using the unbiased rule the probability of refuting a false hypothesis must be no less than the probability of refuting a correct hypothesis. If $\beta_\phi(\theta) > \alpha$ when $\theta \in \Omega_k$, the rule is said to be strictly unbiased, and the rule, the average power of which $\geq \alpha$ in the vicinity of the hypothesis H, is called the "locally unbiased type M" (according to Krishnan). In the single alternative problems of classification, the loss function is usually constant in the sets Ω_H and Ω_k, and the space of the decisions is made up of two elements; therefore the notation (4), as it is easy to see, follows from (3) as a special case.

For the applications, the uniformly most powerful (RNM) rules having maximum probability of correct decision in the entire region of the hypothesis K is of special interest among the unbiased rules. This important property of the RNM unbiased (RNMN) rules completely corresponds to the desired property which the classification system or the system for estimating the parameters with a priori indeterminacy must have. Let us note that the RNM rule, if it exists, always turns out to be unbiased, for its power cannot be less than the power of the trivial rule $\psi(x) \equiv \alpha$. In the broad class of statistical problems where the RNM rules do not exist (and this frequently occurs), nevertheless there are RNMN rules. This again indicates the expediency of constricting the class of all decision procedures to the class of unbiased procedures and finding optimal rules in it. If the parametric space Ω is equipped with topology in which $\beta_\phi(\theta)$ is a continuous function θ for any decision rule $\phi(x)$ and the common boundary ω of the regions Ω_H and $\Omega_k = \Omega \Omega_H$ are not empty, then finding the RNMN rules is equivalent to finding the RNM rules of dimension α in the class of so-called similar rules with respect to P or ω in which the power function

$$\beta_{\varphi}(\theta) = \alpha \quad \text{for all} \quad \theta \in \omega. \tag{5}$$

The quality (5) for the above enumerated conditions follows directly from (4). The rules are said to be similar with respect to P or ω because if ϕ is a nonrandomized rule and has a critical region S, then S "is similar to the selection space" χ in the sense that the probabilities $P_\theta\{X \in S\}$ and $P_\theta\{X \in \chi\}$ do not depend on $\theta \in \omega$. The transition to the class of such rules facilitates the construction of the RNM rule, for the condition (5) is simpler than the condition (4).

For synthesis of the RNMN rule it is important to establish the existence of statistics B(x) sufficient with respect to the family of distributions $\{P_\theta(x), \theta \in \omega\}$ with the boundary ω and completeness of the family $P_\omega = \{P_\theta(B), \theta \in \omega\}$ of distributions of this statistics ($\omega \subset \Omega$). The necessary and sufficient condition that the statistics B(x) be sufficient for the family $P=\{P_\theta(x), \theta \in \Omega, x \in X\}$ (or sufficient for θ when from the contacts it is clear which Ω we are talking about) is the existence of factorization $\{P_\theta(x) = g_\theta[B(x)]h(x)\}$ where $g_\theta[\cdot]$ is a function which depends on θ, and on x only in terms of B(x), and $h(\cdot)$ is a function which does not depend on θ and is identical for the entire family (the factorization criterion [1]). The transition to sufficient statistics will permit reduction of the observation space consisting in discarding that part of the data which does contain information with respect to the distribution $P_\theta(x)$ and, consequently, it is useless for any decision problem pertaining to θ. The formula $P = \{P_\theta(B), \theta \in \Omega\}$ of distributions of the sufficient statistics B(x) is called complete if for any integrable function f(B) the condition $E_\theta[f(B)] = 0$ for all $\theta \in \Omega$ implies f(B) = BP -- almost everywhere (if f(B) is a limited integrable function, then the family P is a limited integral function). For example, the exponential family $\mathcal{P}_\omega = \{P_\theta(B) = C(\theta)h(B)\exp(\sum_{i=1}^{k}\theta_i B_i), \theta \in \omega \subset \Omega\}$ is complete when set ω contains a k-dimensional interval (the theorem of completeness [1]).

Under the condition of existence of the statistic B(x) sufficient with respect to the family $\{P_\theta(x), \theta \in \Omega\}$ and completeness of the family P_ω of distributions of this statistic all such rules have Neuman structure with respect to B, that is, they satisfy the condition:

$$E_\theta[\varphi(x)/B(x)] = \alpha \quad \text{for all} \quad \theta \in \omega. \tag{6}$$

almost everywhere with respect to the family P_ω on the boundary ω. For the Neuman structure rules, constancy of the error probability of the first type on each surface $B(x)$ = const is characteristic. Therefore the problem of finding the optimal rules can be solved on any such surface separately. For this purpose it is necessary simply to proceed to the provisional distributions $P(x/B)$ which do not depend on $\theta \in \omega$ as a result of sufficiency of $B(x)$ with respect to $\theta \in \omega$, which somewhat simplifies the optimization problem (according to the condition (6)). The Neuman structure rules are especially useful and constructive in the presence of the families $P = \{P_{\theta,v}(x)\}$ containing the one-dimensional useful θ and the multidimensional interfering $v = (v_1, \ldots, v_k)$ parameters and in the presence of the corresponding sufficient statistics $T(x)$ and $U(x) = (U_1, \ldots, U_k)$. When using these rules it is possible to exclude the effect of the interfering parameter, for the provisional distribution $P_\theta(T/U)$ does not depend on v on the basis of sufficiency with respect to it of the statistics $U(x)$. The statistic $U(x)$ is sufficient also for the family $\{P_{\theta,\vartheta}(T,U),(\theta,\vartheta)\in\omega\}$ of distributions on the boundary ω, for θ on ω has a fixed value, for example, θ_0 (the hypotheses of the type $H_1:\theta=\theta_0$, $H_2:\theta\leq\theta_0$, $H_3:\theta\geq\theta_0$), and the indicated family completely depends on v. After excluding the interfering parameter it remains to find the rule RNM only with respect to θ. The power function of this rule

$$\beta_\varphi(\theta,\vartheta)=E_\vartheta\{E_\theta[\varphi(x)/U]\} \tag{7}$$

is maximal in the entire range of the hypothesis k.

If the likelihood ratio $P_{\theta'}(T/U)/P_{\theta''}(T/U)$ for any $\theta' \neq \theta''$ is monotonic with respect to T (that is, for any θ' and θ'' the distributions $P_{\theta'}$ and $P_{\theta''}$ are different, and their ratio $P_{\theta'}(T/U)/P_{\theta'}(T/U)$ is a decreasing function of T), and the parameter θ is uniform, the RNM rule coincides with the optimal rule (by the Neuman-Pearson criterion) for checking the simple hypothesis $H_1:\theta=\theta_0$ with respect to the simple alternative $K_1':\theta=\theta_1$ (θ_1 is any value of the parameter from Ω_k), and it has the form

$$\varphi(T,U)=\begin{cases}1 & \text{for } T>C(U),\\ \delta_0(U) & \text{for } T=C(U),\\ 0 & \text{for } T<C(U),\end{cases} \tag{8}$$

where the threshold function $C(U)$ and the probability $\delta_0(U)$ are determined from the condition

$$E_{\theta_0}[\varphi(T,U)/U]=\alpha \text{ for all } U. \tag{9}$$

In view of the fact that the randomized rule (8) does not depend on the specific value of the parameter θ_1, it is suitable for any $\theta \in \Omega_k$, and it is therefore the RNM in the region of the hypothesis K. For $\theta \in \Omega_H$, in view of the monotonicity of the likelihood ratio the power function of the rule (8) $\leq \alpha$, which is simple to demonstrate. Consequently, rule (8) satisfies the conditions (4), and therefore is RNMN. It is significant that the diffusion function of the rule does not depend on the unknown distribution parameters of the observed sample (the value of θ_0 belongs to the boundary ω and therefore is a priori known), and the first type error probability defined by the equality (9) is invariant for any $\theta \in \Omega_H$. Thus, the rule (8), (9) satisfies all of the requirements of practical importance imposed on it, permitting automation of the information processing.

In the applied problems frequently it is necessary to deal with planes of the exponential type (with respect to some sigma-finite dimension) of the type $\{C(\theta,\vartheta)\exp[\theta T(x)+\sum_{i=1}^{k}\vartheta_i U_i(x)]\}$, $\vartheta=(\vartheta_1,\ldots,\vartheta_k)$, $U=(U_1,\ldots,U_k)$. Here the following hypotheses are most frequently considered with respect to the useful parameter $\theta(-\infty<\vartheta<\infty)$: $H_1: \theta=\theta_0$, $K_1: \theta\neq\theta_0$; H_2 : $\theta\leq\theta_0$, K_2: $\theta>\theta_0$; H_3: $\theta\geq\theta_0$, $K_3: \theta<\theta_0$; $H_4: \theta\leq\theta_1$ or $\theta\geq\theta_2$, $K_4: \theta_1<\theta<\theta_2$; $H_5: \theta_1\leq\theta\leq\theta_2$, K_5: $\theta<\theta_1$ or $\theta>\theta_2$. If we realize reduction with respect to the statistic V = h(T, U), then in terms of this statistic (for monotonicity of V with respect to T for each fixed U and independence of V with respect to U for $\theta = \theta_1$ and $\theta = \theta_2$) the RNMN decision function ϕ_5 for the test H_5 is given by the formula (4) on page 219 in [5], and C and δ are found from the equation: $E_{\theta_1}[\varphi_5(V)]=E_{\theta_2}[\varphi_5(V)]=\alpha$. For the test H_4 the RNMN decision function $\phi_4(V)$ is equal to [1]: $\varphi_4(V;\alpha)=1-\varphi_5(V;1-\alpha)$.

3. Concluding Remarks. The use of the principles of unbias and similarity is convenient when it is necessary to obtain optimal (with respect to the Neuman-Pearson criterion) rules for making a decision satisfying the requirements enumerated in item 2. In the known sense the unbias and similarity (the latter is a special case of unbias) are the alternative (with respect to invariance) formalization of the concept of impartiality of the decision procedures. If the principle of invariance is used only when the statistical problem reveals some symmetry, then for application of the principle of unbias there is no such clearly expressed attribute. However, if the decision procedure must satisfy the requirements enumerated in item 2, it is necessary to try to use the unbias and similarity arguments. If the investigated problem is symmetric, then it is necessary to use the principle of invariance giving additional properties that are desirable for the decision making system under the conditions of a priori indeterminacy. Frequently wherever the application of the invariance principle does not lead to success, it is necessary to try to use the unbias principle, imposing defined conditions (4) of restriction on the power function of the decision rule.

The critical regions of the unbias (similar) decision rules in the general case have the form:

$$T(x) > C[U(x)], \tag{10}$$

where T(x) is the statistic sufficient for the family of distributions of the observed selection x of the hypothesis H, U(x) is some statistic, C[·] is the threshold function.

Let us formulate the conditions on satisfaction of which the synthesis of the RNMN decision selection rule is possible.

1. In the distribution of the observed section directly or after some conversion, a one-dimensional useful θ and multidimensional interfering parameter $v = (v_1, \ldots, v_k)$ must be isolated. The parametric space Ω must be convex with the dimensionality k + 1, that is, it must not contain uniformity in any linear space < k + 1.

2. For the useful and interfering parameters it is necessary to realize the sufficient statistics T, $U = (U_1, \ldots, U_k)$.

3. The family of distributions of the statistic U(x) must be complete for $(\theta, v) \in \omega$.

4. The continuity of the power function β_ϕ θ, v) with respect to θ and v is required.

5. The likelihood ratio $P_{\theta''}(T/U)/P_{\theta'}(T/U)$ must be monotonic with respect to T.

Out of these conditions, as is pointed out in [5], the most limiting is the condition of uniformity of the useful parameter, since for many useful parameters there are no regular methods of optimizing the unbiased decision rules, and probably a new optimalness criterion differing from the RNM is required.

As a rule, the remaining conditions are not so burdensome in the problems of classifying and estimating the signal parameters. In particular, they are completely satisfied if the distributions $P_{\theta v}(T, U)$ form an exponential family with uniform useful parameter which is characteristic for the problems of classification and estimation of the parameters of sonar signals.

From the enumerated conditions the following operations follow which are required for synthesis of the optimal unbiased rule.

1. The recording of the distribution of any selection.

2. The determination of the useful θ and interfering v parameters in the distribution of the observed sample.

3. The realization of reduction of the sample space by determination (with respect to the factorization criterion) of statistics T and U which are sufficient with respect to θ and v.

4. Formulation of the hypothesies in the terms of the parameters θ and v.

5. The recording of the joint distribution $P_{\theta,v}(T, U)$ of the statistics T and U and the distributions P(U) and P(T/U).

6. Testing the distribution $P_{\theta,v}(U)$ for completeness on the boundary ω.

7. Testing the power function $\beta_{\phi}(\theta,v)$ for continuity with respect to θ and v.

8. Determination of the monotonicity with respect to T of the plausibility ratio $P_{\theta''}(T/U)/P_{\theta'}(T/U)$ (by the sign of the first derivative).

9. Determination of the decision function $\phi(T, U)$.

10. Determination of the probability $\delta_{\theta}(U)$ and the threshold function C(U).

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JOINT USE OF THE UNBIAS AND INVARIANCE PRINCIPLES IN THE PROBLEMS OF CLASSIFYING AND ESTIMATING PARAMETERS WITH A PRIORI INDETERMINACY

By Yu. Ye. Sidorov, pp 118-120

In order to solve the problem of optimization of the devices for classifying and estimating the signal parameters under the conditions of a priori indeterminacy, it is possible to use the principles of nonbias and invariance [1]. The use of these principles is expedient when it is not possible to find the optimal decision rules in a broad class of rules. In these cases it is necessary to limit ourselves to a narrower class inside which it is possible to hope to find the decision procedures with uniformly least risk. Two of these restrictions are also nonbias and invariance. The principle of nonbias imposes defined restrictions on the power function of the decision rule, and the principle of invariance is applicable if the investigated statistical problem has symmetry, the mathematical expression of which is the invariance with respect to some similar group of transformations. In a number of problems of practical importance in the classification and evaluation of the signal parameters, in particular, sonar, it is possible to use both principles with identical success. According to theorem 6 of Chapter 6 of the monograph [1] this coincidence of the two optimal procedures is not random and occurs when there is a uniformly most powerful (RNM) unbiased (RNMN) rule for the problem of checking the hypotheses which is unique with accuracy to the sets of dimension zero, and there is an RMN rule almost invariant with respect to some group of transformations. In this sense the principles of unbias and invariance are matched. An important conclusion from the mentioned theorem 6 in [1] which is important for applications is the establishment of uniqueness (with accuracy to the sets of dimensions zero) of the RNM invariant (RNMN) rule. The fact is that each RNMN rule has the property of admissibility [1], that is, another rule cannot exist which would be less powerful, and for some alternatives even more powerful than the existing one. This property determines the uniqueness of the RNMN rule. An analogous property for the RNMN rules can also be automatically not satisfied [1], and it must be established for each specific case. The principles of unbias and invariance can also be used as applied one to the other in the following situations: a) when each of them individually does not lead to synthesis of the optimal rule, and with joint application of them this goal is achieved; b) when the unbias rule obtained must have some desirable invariant properties making it still more resistant to changes in the external conditions or, vice versa, the invariant rule must satisfy the unbias conditions; c) when the rule obtained is the RNM among all of the unbiased and invariant rules, but it is unknown whether it will be, for example, RNMN without invariance restrictions. Let us present the conditions of the joint use of the unbias principle and the invariance principle to study the optimal decision rules.

1. The nature of the statistical problem and the requirements on the decision procedure must be such that (see [1, 3, 4]) the application of the unbias and invariance principles is possible.

2. It is necessary to determine the form of the a priori indeterminacy (parametric or nonparametric).

3. It is necessary to satisfy the conditions under which synthesis of the RNMN [3] and RNMN [4] rules is possible (depending on the type of a priori indeterminacy).

The first and third are the most important among these conditions.

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ROLE OF NONPARAMETRIC METHODS IN STATISTICAL HYDROACOUSTICS

By V. V. Ol'shevokiy, F. P. Tarasenko, pp 120-123

1. Introduction. In its modern interpretation [1, 2] statistical hydroacoustics has both common and specific features with other statistical theories of different physical phenomena. The specific nature of statistical hydroacoustics is connected with great complexity and a low degree of study of the ocean as a whole and the hydrophysical phenomena in it determining the hydroacoustic laws. The purpose of this paper is an indication of the place of the nonparametric methods of statistics in the various hydroacoustic columns and discussion of the possibilities which these methods make available. Let us confirm that in many hydroacoustic problems the nonparametric approach is not simply one of possible approaches, but in practice has no alternative. This situation is a natural consequence of the fact that in the nonparametric statement of any statistical problems the functional form of the distribution of the observed variables is considered unknown, and the classes of these distributions are quite broad and are characterized only by the most general information about their properties. It is true that the modern state of the art with respect to the nonparametric statistics far from always satisfies the requirements of hydroacoustics, which, in turn, should stimulate the development of the nonparametric theory itself.

Let us consider the problems of statistical hydroacoustics for which the use of the nonparametric methods appears to be necessary, useful and expedient.

2. The planning of the hydroacoustic experiments. Unfortunately, it must be stated that the results of many years of investigations of hydrophysical characteristics of the ocean rarely succeeds to be fully used when developing the acoustic model of the ocean. This is connected primarily obviously with the fact that when planning the oceanological experiments the researchers have stated limited goals for themselves which do not have in mind the hydroacoustic role of the investigated phenomena. The fact is that any hydroacoustic factor, even one that has been investigated in great detail in itself still does not determine the hydroacoustic effects which depend on the set of many hydrophysical factors. Therefore the creation of the statistical acoustic model of the ocean at the present time has encountered

significant difficulties [4]. As a result, the necessity arises for planning complex oceanological research for hydroacoustic purposes. Considering the high cost of such measurements it is expedient to limit them with the application of the modern methods of statistical experimental planning. The specific peculiarities of statistical hydroacoustics noted above advance the new problem of the development of the methods of planning oceanological experiments and the nonparametric statement. Here, we are dealing with the modification of the standard methods of experimental planning in the proposition of a priori unknown nature of the investigated factors and the observed variables. In particular, let us note that the nonparametric statement of the problems of planning experiments in statistical hydroacoustics leads to reexamination of a number of methods of this theory (formulation of the quality index of the experiments, the types of plants, and so on).

3. Processing of the experimental data. The methods of statistical processing of experimental data essentially are based on the hypothesis of the probability distributions of the investigated values. Instead of limiting ourselves to the qualitative description of the experimental results or construction of the "heuristic" procedures for processing the observations, the experiments must turn attention to the existence of nonparametric algorithms [3] which frequently are not inferior to the classical parametric algorithms with respect to the effectiveness, but they are applicable to a much broader class of experimental situations. Here, however, it is necessary to consider that during the processing, in particular, of nonstationary processes, the nonparametric methods were developed predominately for the cases of the regression models. In this respect the methods of nonparametric estimation of the distributions, in particular, the provisional probability distributions appear to be prospective.

Among the nonparametric procedures for processing the experimental data, the so-called robust stable procedures developed in recent years attract attention. Their attractiveness for statistical hydroacoustics consists in the fact that quality of the statistical conclusions derived by them depends much more weakly on the distribution variations of the input data than in the ordinary, even nonparametric procedures. The concept of stability of the procedures is connected with the necessity for counteraction of the distortions of the experimental data such as the "spoiling" of the selection, the rounding of the values (grouping), the limited nature of the range of the measuring instrument or the time of existence of the observed phenomenon (censure), and so on.

4. The construction of the hydroacoustic mathematical models. The mathematical models in statistical hydroacoustics are used primarily for the following purposes;

A. Prediction of the consequences with respect to the given causes (for example, determination of the characteristics of the hydroacoustic phenomena by the given hydrophysical factors).

B. Quantitative interpretation of the causes by the given consequences (for example, determination of the hydroacoustical factors by the given characteristics of the hydroacoustic signals),

C. Synthesis of hydroacoustic measuring systems,

D. Determination of the operating characteristics of the hydroacoustic measuring systems.

The natural trend in the construction of the most complete possible probability hydroacoustic models for the solution of these problems encounters a deficiency of a priori information which frequently an effort is made to overcome by the weakly substantiated propositions. In addition, the nonparametric approach permits achievement of effective results without calling on the thought-out propositions although, for example, the place and the possibilities of the nonparametric methods in each of the enumerated problems are different. When solving the first two problems (A and B) the nonparametric models can hardly give sufficiently defined quantitative conclusions, although it is excluded that they will turn out to be useful for certain special statements of these problems. The application of the nonparametric methods when synthesizing the receivers of the hydroacoustic signals (problem C) appears to be the most effective. The nonparametric models play the most modest role in the solution of the problems of analyzing the hydroacoustic systems (problem D): the nonparametric methods usually provide judgments of the type of inequalities, estimation of the limits or judgment of the extremalness of one characteristic or another without indication of its numerical value [5].

5. The statistical simulation of the hydroacoustic phenomena and systems. In hydroacoustics, just as in many other areas, it is necessary to resort to statistical simulation on a computer for the solution of the problems of simulation experiments with models of the conditions and measuring systems. During the course of these machine experiments, the following problems arise:

A. Determination of the distributions or their parameters with respect to random samples.

B. Generation of random samples from the given model distributions.

C. Determination of the value for the extremal values of different (frequently multidimensional) functionals not subject to calculation by analytical methods.

It is possible to state that the nonparametric methods can be widely used in all three of the indicated types of problems and, unconditionally, will permit us to obtain effective results.

In addition to a number of already widely used nonparametric procedures for solving the given problems, the authors turn attention to the advantages of using the nonparametric estimate of the probability density with the help of the polygrams which it is expedient to realize in all of the enumerated columns (A, B and C). The properties of the polygrams were investigated in

[3] and, in more detail, in [6]. The main advantage of the polygrams, in addition to the computational simplicity, is the "naturalness"; for example, in contrast to the histogram, the magnitude of the grouping intervals is given by the sample itself. It is true that in the multidimensional case the construction of the polygram is ambiguous, and it is an interesting problem to discover the predominant systems of ordering functions for various problems.

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SIGNAL PROCESSING WITH INCOMPLETE DESCRIPTION OF THE SIGNALS AND INTERFERENCE

By E. A. Dekalo, V. P. Ryzhov, pp 123-125

In this paper the attention given some qualitative characteristics of signal processing with incomplete description of signals and interference is accented. The sharp increase in the variety of the models of the input process is as the volume of a priori information decreases is characteristic. The class of algorithms for processing the processes with incomplete description of them has been investigated highly incompletely. Here the processing quality functional, as a rule, is sensitive to a greater degree to the changes in the signal characteristics than the noise characteristics. In view of the absence of complete data on the characteristics of the input processes it is possible to state the proposition that the synthesis of the optimal systems based on the likelihood ratio with an incomplete description of the input processes is based on the assignment to the processes of sufficiently arbitrary models. In practical situations it appears expedient to express the likelihood ratio in terms of the given incomplete descriptions of the processes: $\lambda = F[\{\Phi_m(\frac{x}{s})\}_{m=1}^{M}]/F[\{\Phi_m(\frac{x}{0})\}_{m=1}^{M}]$ where F[·] is the processing functional; $\{\Phi_m(x/s)\}_{m=1}^{M}, \{\Phi_m(x/0)\}_{m=1}^{M}$ is the set of given provisional functionals characterizing the input process in the presence and absence of a signal.

The incompleteness of the description of the process makes the problem of the resistance of the processing results to variations of the process models especially urgent. Therefore the quality criterion of the signal processing system must include the noiseproofness characteristics in the sensitivity of the system to the variations of the signals and the interference. If we denote the characteristic of the noiseproofness by the simple Q and the possible conditions of the functioning of the system by the symbol R, then the quality criterion of the system can be representable in the form $K = \int_{\Omega_R} Q(R)W(R)dR$ or be determined by the conditions $Q = Q_{max}$ for $R \in \Omega_R$, $\Delta R = \Delta R_{max}$ for $\theta \geq \theta_0$.

As the sensitivity characteristics of the system, the functionals of extent can be used for variations of the signal interference characteristics, the variation of the distribution entropy, the limits of variation of the random parameters, and so on. In some cases the condition of functional stability is defining when synthesizing the signal processing systems. As an example let us present the algorithm for signal detection with monotonic intrapulse frequency modulation invariant to the doppler transformation of the signal spectrum [1]. The algorithm is based on comparing the durations of two adjacent intervals between the zeros of the process, the discovery of the partial solution of the presence of the signal for each comparison and accumulation of the partial solutions in digital form. For the broad class of interference the probability of the positive sign of the difference of the duration of the adjacent interval is equal to 0.5; for the signal and interference set it is greater than 0.5 (these probabilities are defined in terms of the probability density of the instantaneous frequency of the signal and interference set). As follows from the essence of the algorithm, the signal transformations for which monotonicity of the dependence of the instantaneous frequency on time maintained have little influence on the noiseproofness of the investigated detector.

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UDC 621.391.2

NONPARAMETRIC CLASSIFICATION OF NARROW-BAND HYDROACOUSTIC SIGNALS

By K. T. Protasov, pp 125-126

The restrictions on the class of signals characterizing the recognized hydroacoustic situation will permit "simplification" of the decision rule of the type of the likelihood ratio constructed on the nonparametric estimates of the probability density [1]. The structure of the nonparametric estimates of the vector density $x = (x_1, \ldots, x_k)$, $x \in X^k$ using the sample values $\| \xi^s_{ij} \|$ $j = 1$ to k, $s = 1$ to 2, $i = 1$ to n_s, where s is the number of the class, k is the dimensionality of the space X^k of the observations of the vector of the random variables (the readings of the random process at discrete points in time $(0 \leq t_1, \ldots, t_k \leq T < \infty)$, ... [symbol missing] is the volume of the samples from the class, has the form [2]:

$$\tilde{\rho}_s(x) = \frac{1}{n_s} \sum_{i=1}^{n_s} \frac{1}{(2\pi)^{k/2} |R|^{1/2} h_s^k} \exp\left\{-\frac{1}{2h_s^2} (x - \xi_i^s)^T R^{-1} (x - \xi_i^s)\right\}, \tag{1}$$

where R is the positively defined matrix. The estimates (1) are meaningful on satisfying the conditions [2]: $h_s \to 0$, $n_s (h_s)^k \to \infty$ for $n_s \to \infty$. For determinacy we shall consider the stucture of the decision rule of the type:

$$\tilde{\ell}(x) = c\left(\ln \frac{\tilde{\rho}_1(x)}{\tilde{\rho}_2(x)} - \ln(t)\right), \tag{2}$$

where $c(y) = \{1, y \geq 0; 0, y < 0\}$, t is the corresponding threshold. In order to improve the properties of the estimates it is expedient to assign to the matrix the index of the class number R_s. Then we shall set R_s as the correlation matrix of the vector x, which gives the estimate (1) which is optimal in the mean square sense for the distributions of the x type: $|R_s|^{-1/2} \varphi(x^T R_s^{-1} x)$ [2]. For the matrices R_s and R_s^{-1}, the following expansions are valid:

$$R_s = \Phi_s^T \Lambda_s \Phi_s, \quad R_s^{-1} = \Phi_s^T \Lambda_s^{-1} \Phi_s, \quad \Phi_s^T \Phi_s = I, \quad |R_s| = |\Lambda_s| = \prod_{j=1}^{k} \lambda_j^s, \tag{3}$$

where Φ_s is the matrix of orthonormalized transformation (the discrete version of the Karunev-Loev expansion), Λ_s is the diagonal matrix of eigenvalues. Let us use the approximation of quadratic form of the type:

$$Q = \Delta^T \Lambda^{-1} \Delta = \sum_{j=1}^{k} \frac{\Delta_j^2}{\lambda_j} = \sum_{j=1}^{m} \frac{\Delta_j^2}{\lambda_j} + \sum_{j=m+1}^{k} \frac{\Delta_j^2}{\lambda_j}, \quad \Delta_j = (c_j - c_j(\cdot)), \tag{4}$$

which is based on the following experimental facts: if the eigenvalues λ_j are arranged in decreasing order, then in a number of practical problems of hydroacoustics, a sharp decrease in the values of λ_j is observed for $j \leq m$, where m is a small number by comparison with k, after which the spectrum of the eigenvalues is equalized slowly, decreasing (the processes with such a property will be defined as "narrow band").

The problem of selecting the number m in (4) is solved when investigating the spectrum of the eigenvalues and the information about the accuracy of recording the initial data. The decision rule (2), (considering (4), will be represented as follows after transformation

$$\hat{\ell}(x) = c\{\ln(\sum_{i=1}^{n_1} \exp[-\frac{1}{2h_1^2}[\sum_{j=1}^{m_1+1} \nabla_{ij}^1]\})/(\sum_{i=1}^{n_2} \exp[-\frac{1}{2h_2^2}[\sum_{j=1}^{m_2+1} \nabla_{ij}^2]\}) - \beta\}, \tag{5}$$

where

$$\nabla_{ij}^s = (c_j^s - c_{ji}^s)^2/\lambda_j^s, \; j = 1 \div m_s, \; \nabla_{i,m_s+1} = \varepsilon_{si}^2/\bar{\lambda}^s, \; s = 1 \div 2, \; \beta = \ln t + \ln(n_1/n_2) + \frac{1}{2}\ln(\prod_{i=1}^{m_1} \lambda_i^1 / \prod_{i=1}^{m_2} \lambda_i^2) + \frac{1}{2}\ln[(\bar{\lambda}^1)^{k-m_1}/(\bar{\lambda}^2)^{k-m_2}] + k\ln(h_1/h_2).$$

The values of ε_{si}^2 considering that: 1) $k \gg m$, 2) the vectors x and ξ_i are statistically independent, 3) the coefficient c_j and c_ℓ, c_{ji} and $c_{\ell i}$ for $j \neq \ell$ are uncorrelated, 4) $M[c_j^2] = M[c_{ji}^2] = \bar{\lambda}$, $j = (m+1) \div k_{m'}$ to 5) $M[c_j] = M[c_{ji}] = 0$ then the value $2\Sigma c_j c_{ji}$, have the form $\varepsilon_i^2 \approx (E - \sum_{j=1}^{m} c_j^2) + (E_i - \sum_{j=1}^{\ell} c_{ji}^2)$.

Thus, the decision rule $\hat{\ell}(x)$ contains the vectors having the dimensionality $(m_s + 1) \ll k$, which permits investigation of the structure of the recognition device in the form of two modules: in the first of them the spectral components are calculated which must be called the attributes (the receptor module) and in the second, the decision rule is realized (5) (the decision module).

Let us now consider the problems of training the recognition algorithm, that is, determination of the unknown parameters of the decision rule (5). The number of readings k of the random processes selected, as usual, from the conditions of the Kotel'nikov theorem. The parameters of diffuseness of the nonparametric estimates h_1 and h_2 are determined from the condition of the minimum empirical risk estimated by the training sample [1]. It is possible to determine Φ knowing the correlation matrices R; the latter are estimated using the initial samples $\| \xi_{ij} \|$, i = 1 to n, j = 1 to k. Here the rank of the estimate of the correlation matrix is no greater than min(n, k). In order to construct m attributes, we only need m rows of the matrix Φ corresponding to the small eigenvalues λ which can be obtained by the algorithm for constructing the adapted base [1]. The number m < min(n, k) which is also natural, for m is determined by the accuracy of the approximation of the initial realizations, and not from consideration of the values of n and k which do not have direct bearing on the number of attributes.

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UDC 62-501

POSSIBILITY OF THE APPLICATION OF NONPARAMETRIC ESTIMATES IN THE PROBLEMS OF DETECTION AND CLASSIFICATION

By G. M. Koshkin, pp 127-128

1. Let it be necessary to check the following hypothesis by the random sample $x_1, \ldots, x_n$ from realization of a mixture of signal and noise: all x_i have probability density of the interference $f(z)$ as opposed to the alternative: each x_i has the density $f(z_i - \gamma s_i/\sqrt{n})$ of the mixture of the signal $\gamma s(t)/\sqrt{n}$ and additive noise $\xi(t)$, where $s(t)$ is the given function, γ is a constant, $S_i = S(t_i)$, $x_i = x(t_i)$. Then the asymptotically optimal algorithm for detecting the deterministic signal $\gamma S(t)/\sqrt{n}$ will permit the decision to be made regarding its presence with the probability of false alarm α [1], if $n^{-1/2}\sum_{i=1}^{n} S_i \varphi(x_i) > x_\alpha\sqrt{QI}$, where $\varphi(x) = -f'_x(x)/f(x)$, $I = \int \varphi^2(x) f(x)\,dx$, $Q = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} S_i^2$, x_α is the normal distribution percentage point.

Theorem 1. If $\xi_1, \ldots, \xi_n$ is a sample of interference $\xi(t)$ with density $f(x)$ unknown to us, having limited derivatives

$$f'_x(x), f''_{x^2}(x), f'''_{x^3}(x), f''''_{x^4}(x); \lim_{n\to\infty}(h_n + n^{-1}h_n^{-3}) = 0; 0 \leq K(u) < \infty, K(u) = K(-u), \int u^2 K(u)\,du = 1, \lim |u^3 K(u)| = 0, \int [K'(u)]^2 du = M, \int K^2(u)\,du < \infty,$$

then for $n \to \infty$ the estimate

$$\varphi_n(x) = \sum_{i=1}^{n} K'\left(\frac{x-\xi_i}{h_n}\right) / h_n \sum_{i=1}^{n} K\left(\frac{x-\xi_i}{h_n}\right)$$

converges in the mean square to $\phi(x)$, where $u^2(\varphi_n) = E[\varphi_n(x) - \varphi(x)]^2 \cong Mf(x)/nh_n^3 + h^4[f'''_{x^3}(x)f(x) - f''_{x^2}(x)f'_x(x)]^2/4f(x)$, and the optimal h_n^0 minimizing $u^2(\varphi_n)$ has the form:

$$h_n^0 = [3Mf^2(x)/n[f'''_{x^3}(x)f(x) - f''_{x^2}(x)f'_x(x)]^2]^{1/7}.$$

As the $\sqrt{n}$-meaningful estimate I let us take [2]:

$$I_n = \frac{1}{n}\sum_{j=1}^{n}\left[\sum_{i=1}^{n}K\left(\frac{\xi_j-\xi_i}{h_n}\right)\right]^2\left[h_n\sum_{t=1}^{n}K\left(\frac{\xi_j-\xi_t}{h_n}\right)\right]^{-2}.$$

2. Let $\vec{x}_1^{\ell}$, $\vec{x}_2^{\ell}$, ..., be the sequence of independent and identically distributed k-dimensional random vectors characterized by the densities $f_\ell(\vec{x})$, $(\vec{x}) = (x^1, \ldots, x^k)$, ℓ be the class number, $\ell = 1, 2$, where $P_r\{\vec{x}=\vec{x}_i^1\} = q_1$, and $P_r\{\vec{x}=\vec{x}_i^2\} = 1-q_1$, $i=1,2,\ldots$. For the decision procedure $D(\vec{x}) = q_1 f_1(\vec{x}) - (1-q_1)f_2(\vec{x})$ let us consider the recurrent training algorithm [3, 4]:

$$D_n = D_{n-1} - \frac{1}{n}\left\{D_{n-1} - h_n^{-k}\left[\rho_n\prod_{j=1}^{k}K\left(\frac{x^j - x_n^{j1}}{h_n}\right) - (1-\rho_n)\prod_{j=1}^{k}\left(\frac{x^j - x_n^{j2}}{h_n}\right)\right]\right\},$$

where $P_n = 1$ if we observe $\vec{x}_n^1$, $P_n = 0$, if we observe $\vec{x}_n^2$.

Theorem 2. If $D(\vec{x})$ satisfies the expansion R [4]: $0 \le K(u) < \infty$

$$K(u)=K(-u),\ \int K(u)du=1,\ \int u^2K(u)du=1,\ \int K^2(u)du=L;\ h_n = An^{-\alpha},$$
$$0<k\alpha<1/2,\ A>0;\ \int\ldots\int\sum_{\ell=1}^{k}(u)^{1+\alpha/2}\prod_{\ell=1}^{k}K^{2+\delta}(u_\ell)du^1\ldots du^k<\infty,$$
$$\alpha>0,\ \delta>0 \text{ then for } n\to\infty:$$

1/ $u^2(D_n) \cong \frac{L^k D(\vec{x})}{(1+k\alpha)^2 n h_n^k} + \frac{h_n^4}{4(1-2\alpha)^2}\left[\sum_{\ell=1}^{k}\frac{\partial^2 D(\vec{x})}{\partial(x^\ell)^2}\right]^2$;

2/ $\sqrt{nh_n^k}\,[D_n - D(\vec{x})]$ converges with respect to the distribution to the normal law $N(0, D(\vec{x})(1+k\alpha)^{-1}L_k)$;

3/ the optimal h_n° minimizing $u^2(D_n)$ have the form:

$$h_n^\circ = \left[\frac{k(k+2)L^k D(\vec{x})}{2(k+4)n}\left[\sum_{\ell=1}^{k}\frac{\partial^2 D(\vec{x})}{\partial(x^\ell)^2}\right]^{-2}\right]^{\frac{1}{k+4}}.$$

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UDC 621.391.2

EXPERIMENTAL STUDIES OF RANKED ALGORITHMS FOR PROCESSING HYDROACOUSTIC SIGNALS

By A. Ya. Kalyuzhvyy, L. G. Krasnyy, pp 128-132

Theoretically, ranked signal detection algorithms insure invariance of the level of false alarms to the functional form of the noise distribution only under the condition [1] that the elements of the rank set are independent and are identically distributed. However, in practice on the basis of the nonstationarity, nonunformity and nonwhiteness of the interference field these propositions cannot be satisfied. Hence, the problem arises of experimental testing of the stabilizing capacity of the ranked algorithms under actual conditions. The experimental test unit was assembled for the solution of this problem, the structural diagram of which is presented in Figure 1.

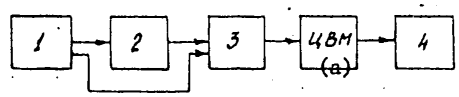

Figure 1. Structural diagram of the experimental test unit.

Key: a. digital computer

In accordance with this scheme, the signals from the output of the tape recorder 1 are fed to the standard receiving channel 2 containing a comb of filters, the envelope detectors and integrators. The output voltages of the receiving channel, comes through the multichannel analog-code converter 3 to the digital computer. The post-detector processing in the computer consisted in accumulating n signal readings (with respect to one of the algorithms indicated below) and decision making about the presence or absence of a signal in each resolving "cell." The results of the processing word depicted in the display 4 for which the ATsPU-128 alphanumeric printer was used. Each position of the alphanumeric printer corresponded to a defined value of the time of arrival of the signal, and one row, to the location cycle. For clarity, the values of the test statistics were compared not with one threshold but with three $P_1 < P_2 < P_3$, corresponding to the probabilities of false alarm $\alpha_0 = 10^{-1}, 10^{-2}, 10^{-3}$). If in some section $T < P_1$, then in the corresponding position the ATsPU [alphanumeric printer] prints out a space; if $P_1 < T < P_2$, the symbol "-"; if $P_2 < T < P_3$, the symbol "*," and in the case of $T > P_3$, the symbol "x." For the selected method of representation the presence of a signal in some band segment was accompanied by the appearance of a characteristic vertical track (Figures 2, 3). The composition of the symbols entering into the track made it possible qualitatively to determine the improvement or worsening of the noiseproofness of one processing algorithm by comparison with the other and the symbol distribution over the working field of the display and their number, the correspondence of the actual false alarm level to the given α_0.

The following types of processing were investigated:

1. Traditional processing. In the jth position of the band in the ith frequency channel the decision is made by the algorithm:

$$\sum_{k=1}^{n} x^2[i+f_k, j+t_k] \gtrless \tilde{\sigma}^2 P_{\alpha_0}, \qquad (1)$$

where x[i, j] is the reading of the voltage in the ith frequency channel at the jth point in time, $\{f_k\}_1^n$ and $\{t_k\}_1^n$ are the frequency and time codes of the signal, $\tilde{\sigma}^2$ is an estimate of the interference dispersion, $P^1_{\alpha_0}$ is the threshold calculated under the assumption that the interference at the channel input has standard gaussian distribution.

2. The nonparametric processing with ranking with respect to time. The processing consists of two steps. In the first step in each frequency channel a sliding ranking of the sample with respect to the algorithm is carried out:

$$R[i,j] = \sum_{k=j-1}^{j-m_1} u(x[i,j]-x[i,k]) + \sum_{k=j+1}^{j+m-m_1} u(x[i,j]-x[i,k]), \qquad (2)$$

where u(z) = 1 for z > 0, u(z) = 0 for z < 0 and u(z) = ξ for z = 0(ξ is a random variable with equal probability of assuming values of 0 or 1). The parameter m_1 determines the position of reckoning x[i, j] in the reference interval containing m samples.

In the second step the decision is made by the algorithm:

$$\sum_{k=1}^{n} h\{R[i+f_k, j+t_k]\} \gtrless \Pi_{\alpha_0}, \qquad (3)$$

where $h(k) = \sum_{i=1}^{k} 1/(m-i+1)$.

3. The nonparametric processing with ranking with respect to frequency. In the first step the joint ranking of the frequency channels is carried out in accordance with the expression:

$$R[i,j] = \sum_{k=1}^{n} u(x[i,j] - x[k,j]). \tag{4}$$

In the second, the decision is made by algorithm (3).

In order to check out the channel and estimate the potential capabilities of the algorithms, before processing the natural recordings a simulation was carried out. The signals and the interference corresponding with respect to their properties to the theoretical model of the situation at the receiver input were formed directly on the digital computer from a set of pseudorandom numbers.

Initially the case was simulated where the interference is stationary and uniform with respect to frequency. As a result, the processing for this model were used as the standard processings when interpreting the experimental data. Then the interference of the type of marine reverberation which is nonstationary with respect to level (see Figure 2) was simulated.

As the model experiment demonstrated, for traditional processing (the upper half of Figure 2) at short distances, continuous lighting of the working field of the display by the false interference marks is observed, which mask the signal track, and at large distances neither the false marks nor the signal track are observed. During nonparametric processing with ranking with respect to time and symmetric positioning of the reference interval (the lower half of Figure 2), the level of false alarms in the same situation corresponds to the rated level and the tracks of the two signals are clearly observed.

Analogous processing was carried out also for natural hydroacoustic signals. As a result, a large volume of experimental data were obtained which permit the following choices to be made:

1. The nonparametric ranked processing under the conditions of real hydroacoustic interference insures a significantly higher degree of stabilization of the false alarm level than traditional. This conclusion is valid both for noise and reverberation channels (the results of the processing in the reverberation channel presented in Figure 3 are fully analogous to the above-presented simulation results).

2. The nonparametric processing with ranking with respect to time actually in all cases insured satisfactory results for $m_1 = m/2$ (the symmetric arrangement of the reference interval). The ranking with respect to frequency is effective only in the noise channels.

3. The stabilization of the level of the false alarms by the nonparametric processing channel in practice did not depend on whether the automatic control system of the primary processing channel is on or off. During traditional processing the automatic control system was switched on only for partial improvement of the stabilization.

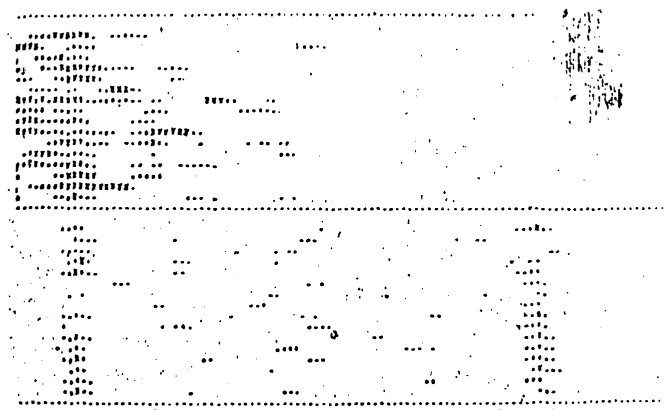

Figure 2.

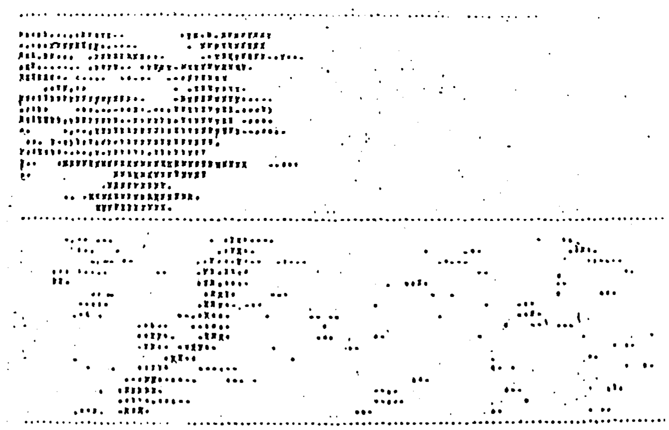

Figure 3.

4. The experiment demonstrated that when connecting the outputs of the frequency channels through the selection system with respect to the maximum (SOM), in each frequency channel it is expedient to perform nonparametric processing in advance with ranking with respect to time. This permits equalization of the interference properties in the various channels.

The nonparametric processing at the SOM output has less effectiveness.

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UDC 519.2

TWO-STEP SEARCH FOR A MOVING TARGET

By A. F. Terpugov, F. A. Shapiro, pp 132-133

A two-step search algorithm for a moving target is proposed which is a version of the well-known search procedure of Ye. Pozner [1] permitting consideration of the possible displacements of the object in the multichannel system from one channel to another. The version consists in the fact that after the preliminary inspection step, the measured values of the logarithm of the likelihood ratio in the form of a linear combination are recalculated, the coefficients of which depend on the mean displacement rate of the target. Then all of the channels are ordered in decreasing order of the enumerated values of the logarithm of the likelihood ratio. A study is made of the situation where during the time between the preliminary and final inspections the target can turn out to be in the same channel as it was with a probability of 1 - P, and it can either go to the next one to the left or to the next one to the right with a probability of P/2. The recalculation of the measured values of the logarithm of the likelihood ratio takes place by the formula

$$u_j' = q u_{j-1} + (1-2q) u_j + q u_{j+1}, \quad 0 \le q \le 1. \tag{1}$$

The investigation of the proposed algorithm is performed under the following assumptions: in all there are N channels (N >> 1), and the object can be present in only one of them. Let us consider that there is also one search device. The logarithm of the likelihood ratio is assumed to be a normal random variable with the dispersion $2\mu t_0$ and mathematical expectation $\pm\mu t_0$ depending on whether there is an object in the channel or not. Here μ is the value characterizing the channel and having the meaning of the ratio of signal to noise, and t_0 is the time for preliminary examination of each channel.

Let $\bar{t}$ be the average detection time for the channel with the object. Then considering the numerical characteristics of the random variable u'_j let us consider the value of $\mu\bar{t}/N$ and let us proceed to the limits for $N \to \infty$. As a result of the simple transformation we obtain:

[redacted] [redacted]

$$\bar{\tau} = \lim_{N\to\infty} \mu \bar{t}/N = \alpha + \alpha_1\left[(1-p)\Phi\left(-\frac{(1-2q)\sqrt{a}}{\sqrt{1-4q+6q^2}}\right) + p\Phi\left(-\frac{q\sqrt{a}}{\sqrt{1-4q+6q^2}}\right)\right], \qquad (2)$$

where $\alpha = \mu t_0,\ \alpha_1 = \mu t_1,\ \Phi(x) = (2\pi)^{-1/2}\int_{-\infty}^{x} e^{-t^2/2}dt.$

The optimal values of the parameters q and c are obtained from the equations $\partial\bar{t}/\partial q = 0$, $\partial\bar{t}/\partial c = 0$. A comparison of the investigated algorithm will be made with the algorithm of [1] which is obtained from (2), for q = 0. In this case the optimal value of the parameter $c = c_1$ is obtained from the equation $\partial\bar{t}_p/\partial c = 0$. The game in the average detection time of the channel with the object is

$$\Delta = (c_1^2 + \alpha_1[(1-p)\Phi(-c_1) + p/2])/(c^2 + \alpha_1[(1-p)\Phi(-(1-2q)c/\sqrt{1-4q+6q^2}) + p\Phi(-qc/\sqrt{1-4q+6q^2})]).$$

In Figure 1 examples are presented of the dependence of Δ on α_1 for various values of p, from which it follows that as a result of optimization with respect to the time of preliminary examination, a reduction in the average detection time is obtained which can reach 60%.

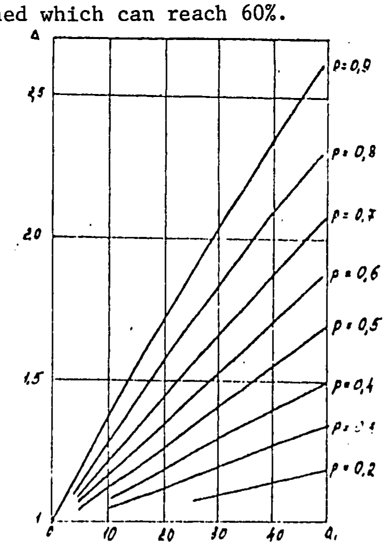

Рис. 1

Figure 1.

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UDC 62-501.12

IDENTIFICATION OF LINEAR SYSTEMS BY THE LEAST FUNCTION METHOD

By K. I. Livshits, N. Yu. Mirgolis, A. F. Terpugov, pp 133-135

It is necessary to deal with the identification of linear systems in hydroacoustics when determining the characteristics of the reverberation interference, the characteristics of the targets and the solutions of the target recognition problem. In the case where the fluctuation noise is normal, the least squares method is the optimal method for this purpose. If nothing is known about the noise statistics, then sometimes it is expedient to use other methods. A discussion is presented below of the basic ideas of identifying the targets by the least functions method which has defined advantages over the least squares method.

After digitalization, the problem of identifying the linear system can be described in the following way. By the observed values of

$$y_i = \sum_{k=1}^{n} c_k \varphi_{ki} + n_i ,$$

where $(i = \overline{1, N})$, $\| \phi_{ki} \|$ is the experimental planning matrix, n_i are the independent random variables with symmetric probability distribution $t(x)$, it is necessary to define the unknown regression coefficients $c_k (k = \overline{1, n})$. The idea of the investigated method consists in the following. Let $\hat{c}_k$ be the estimate of C_k. Then $\hat{n}_i = y_i - \sum_k \hat{C}_k \phi_{ki}$ are the estimates of the ith value of the value of n_i. Let r_p be an estimate of the measure of the dependence of $\hat{n}_i$ and $\hat{n}_{i+p} (i = \overline{1, N-\Gamma})$, that is, the measures of the function in the p-step such that $r_p \geq 0$, that for independent random variables $r_p = 0$. Let us consider the expression

$$f = \sum_{p=1}^{N-1} \alpha_p r_p , \quad \alpha_p > 0 \quad \forall p$$

It is obvious that if $\hat{c}_k$ coincides with c_k, then $\hat{n}_i$ coincides with n_i, and by assumption n_i and n_{i+p} are independent. Therefore for $\hat{c}_k = c_k$ with the

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values of r_p will be near zero and f will be small. If $\hat{c}_k \neq c_k$, then $\hat{n}_i = n_i + \sum_k (c_k - \hat{c}_k)\varphi_{ki}$, as a result of the presence of a second term $\hat{n}_i$ and $\hat{n}_{i+p}$ will be dependent, and f will be greater than zero. Therefore it is proposed that the estimate $\hat{c}_k$ of the variables c_k be found from the condition of minimum f with respect to $\hat{c}_k$ $(k = \overline{1, n})$. It is possible to take the measure of the function r_p in different form. In particular, two types of r_p were investigated; nonlinear generalization of the correlation coefficient where

$$r_p = \left[\frac{1}{N-p}\sum_{i=1}^{N-p} K(\hat{n}_i)K(\hat{n}_{i+p})\right]^2, \tag{1}$$

where $K(\cdot)$ is an odd, monotonically increasing function, and the nonparametric Kendall correlation coefficient where

$$r_p = \left[\frac{1}{(N-p)^2}\sum_{i\neq j} \operatorname{sign}(\hat{n}_i - \hat{n}_j)\operatorname{sign}(\hat{n}_{i+p} - \hat{n}_{j+p})\right]^2. \tag{2}$$

The basic results reduce to the following:

1. If all $\alpha_p > 0$ and $\sum_{p=1}^{\infty} \alpha_p < \infty$, for certain restrictions on the experimental planning matrix, the estimates obtained by the least functions method are asymptotically unbiased and independent.

2. In the case where nonlinear generalization of the correlation coefficient (1) is used, the variation matrix of the estimates of the parameters $\hat{c}_k$ has the following form:

$$V_{\hat{c}_k} = \frac{\int_{-\infty}^{\infty} K(u)^2 g(u)\,du \cdot \int_{-\infty}^{\infty} \dot{K}(u)^2 g(u)\,du}{\left[\int_{-\infty}^{\infty} \dot{K}(u) g(u)\,du\right]^4} M\{B^{-1}A_1B^{-1}\} + \frac{\int_{-\infty}^{\infty} K(u) g(u)\,du}{\left[\int_{-\infty}^{\infty} \dot{K}(u) g(u)\,du\right]^2} M\{B^{-1}(A_2+A_3)B^{-1}\}, \tag{3}$$

where

$$B_{pq} = \sum_k \frac{\alpha_k \rho_k}{N-k} \sum_{i=1}^{N-k} (\varphi_{p,i}\varphi_{q,i+k} + \varphi_{p,i+k}\varphi_{q,i}),\quad A_{1pq} = \sum_k \frac{\alpha_k^2 \rho_k^2}{(N-k)^2} \sum_{i=1}^{N-k} (\varphi_{p,i}\varphi_{q,i} + \varphi_{p,i+k}\varphi_{q,i+k}),$$

$$A_{2pq} = \sum_k \frac{\alpha_k^2 \rho_k^2}{(N-k)^2} \sum_{i=1}^{N-k} (\varphi_{p,i+k}\varphi_{q,i-k} + \varphi_{p,i-k}\varphi_{q,i+k}),$$

$$A_{3pq} = 2\sum_{k<\tau} \frac{\alpha_k \alpha_\tau \rho_k \rho_\tau}{(N-k)(N-\tau)} \sum_{i=1}^{N-k} (\varphi_{p,i+k}\varphi_{q,i+\tau} + \varphi_{p,i-k}\varphi_{q,i+\tau} + \varphi_{p,i+k}\varphi_{q,i-\tau} + \varphi_{p,i-k}\varphi_{q,i-\tau}),$$

and M denotes averaging with respect to independent normal random variables ρ_k having zero means and unit dispersion.

3. In the case where the rank Kendall correlation coefficient (2) is used, the variation matrix of the estimates of the parameters of the form:

$$V_{\hat{c}_k} = \frac{2}{3\left[\int_{-\infty}^{\infty} g(u)^2 du\right]^2} M\{B^{-1}(A_1+A_2)B^{-1}\} + \frac{2\int_{-\infty}^{\infty} g(u)^3 du}{3\left[\int_{-\infty}^{\infty} g(u)^2 du\right]^4} M\{B^{-1}A_3B^{-1}\},$$

where B, A have a structure analogous to (3), but in view of awkwardness, they are not presented here.

4. In order to find estimates of the parameters it is possible to propose an iterative numerical algorithm realizable on a computer.

UDC 621.391.2

CONVERGENCE OF THE PARAMETERS OF AN ADAPTIVE KALMAN FILTER

By L. G. Krasnyy, V. P. Peshkov, pp 135-138

In [1, 2] algorithms are proposed for the adaptive Kalman filtration of the signal with spectral power density $g_S(\omega) = g_S\alpha^2/2(\alpha^2 + \omega^2)$ with unknown parameter α characterizing the signal band against a background of white noise $g_N(\omega) = g_N/2$. The structure of the adaptive filter in this case can be described by the system of stochastic equations [2]:

$$
\begin{aligned}
&S_k^* = (1-\alpha_{k-1}^*\Delta t)S_{k-1}^* - K_{\alpha S_{k-1}}\Delta t + 2/g_N K_{SS_{k-1}}(U_k - S_{k-1}^*)\Delta t,\\
&\alpha_k^* = \alpha_{k-1}^* + 2/g_N K_{\alpha S_{k-1}}(U_k - S_{k-1}^*)\Delta t,\\
&K_{SS_k} = (1-2\alpha_{k-1}^*\Delta t)K_{SS_{k-1}} - 2/g_N K_{SS_{k-1}}^2\Delta t - 2S_{k-1}^* K_{\alpha S_{k-1}}\Delta t +\\
&\qquad + g_s/2\, K_{\alpha\alpha_{k-1}}\Delta t + g_s/2\, \alpha_{k-1}^{*2}\Delta t,\\
&K_{\alpha S_k} = (1-\alpha_{k-1}^*\Delta t)K_{\alpha S_{k-1}} - 2/g_N K_{\alpha S_{k-1}} K_{SS_{k-1}}\Delta t - S_{k-1}^* K_{\alpha\alpha_{k-1}}\Delta t,\\
&K_{\alpha\alpha_k} = K_{\alpha\alpha_{k-1}} - 2/g_N K_{\alpha S_{k-1}}^2\Delta t,
\end{aligned}
\qquad (1)
$$

where S_k^*, α_k^* are estimates of the signal and the parameter α; U_k is the input realization; $S_0^* = \alpha_0^* = K_{SS_0} = K_{\alpha S_0} = 0$; $K_{\alpha\alpha_0} \neq 0$ are the initial conditions. The simulation of the algorithm (1) demonstrated that the magnitude of the established value of the parameter α^* depends on the selected a priori dispersion and for an unsuccessfully selected $K_{\alpha\alpha_0}$ can essentially differ from the true value of α (Figure 1). In order to eliminate this deficiency, in the given paper the iterative approach to the construction of $K_{\alpha\alpha_0}$ is proposed which based on the experimentally established function $K_{\alpha\alpha_0} = f(\alpha)$ (Figure 2).

The indicated procedure consists in the following:

In the first step an arbitrary value of $K_{\alpha\alpha_0}$ is selected;

After a time interval equal to the setup time, an estimate α^* of the parameter α is obtained in the Kalman filter;

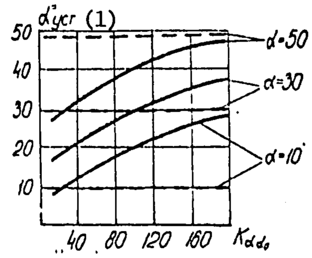

Figure 1.

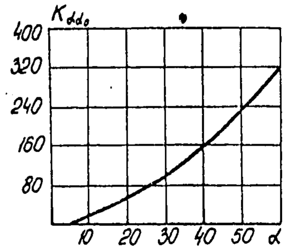

Figure 2.

Key: 1. α^*_{setup}

Using the function $K_{\alpha\alpha_0} = f(\alpha)$, the initial value of $K_{\alpha\alpha_0}$ in equations (1) is more precisely defined;

By the refined estimate of $K_{\alpha\alpha_0}$ the new estimate α^* of the parameter α is determined using the equations (1) and so on.

The convergence of the proposed algorithm is confirmed by simulation of it on a digital computer for various values of the a priori dispersion $K_{\alpha\alpha_0}$. In Figure 1 the dotted lines depict the established values of the estimated parameter α using the proposed iterative procedure according to $K_{\alpha\alpha_0}$; in Figure 3 the dotted line depicts the convergence of the a priori dispersion $K_{\alpha\alpha_0}$ to the required dispersion of their function of the number of filtration cycles.

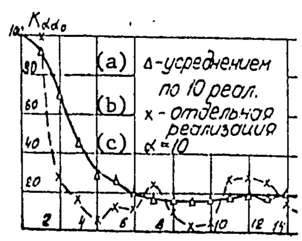

Figure 3.

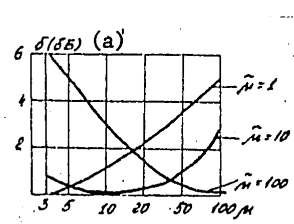

Figure 4.

Key: a. Δ -- averaging with respect to 10 realizations
b. x -- individual realization
c. $\alpha = 10$

Key: a. decibels

The presented results were obtained for a known value of the spectral density g_s. However, as a rule, the spectral density g_s is unknown. In this case it is proposed that the parameter g_s be fixed in the Kalman filter, selecting it within the limits of the possible values of the dynamic range of the signals. Let us demonstrated that in the given case the proposed iterative procedure with the presented control curve $K_{\alpha\alpha_0} = f(\alpha)$ is effective. For this purpose initially we formulate the requirements on the accuracy of giving the parameter g_s in the nonadaptive Kalman filter (for known α), and then we investigate the adaptive filter (for unknown α and g_s) with the iterative procedure with respect to $K_{\alpha\alpha_0}$. It is possible to show that if in the nonadaptive Kalman filter instead of the true value of the spectral density g_s we use a value of $g_{s\phi} \neq g_s$, then the relative mean square error in the signal filtration will be:

$$\delta_t = \frac{\varepsilon_t}{\varepsilon_0} = 1 - \left\{1 - \frac{(\sqrt{1+\tilde{\mu}} - 1)^2 + \mu}{\mu\sqrt{1+\tilde{\mu}}}\right\}\left[1 - \exp(-2\alpha\sqrt{1+\tilde{\mu}}\,t)\right], \tag{2}$$

where $\varepsilon_0 = g_{s\alpha}/4$ (sic) is the signal dispersion; $\mu = g_s/g_N$, $\tilde{\mu} = g_{s\phi}/g_N$.

In Figure 4 we have the graphs for the steady state relative filtration error $\delta = 10 \lg \tilde{\varepsilon}_\infty/\varepsilon_\infty$ of the signal s(t) as a function of the signal/noise ratio μ at the filter input for different values of $\tilde{\mu}$. From the figure it follows that the inexact assignment of g_s in the Kalman filter has a significant influence on the signal filtration errors. The best results (losses in accuracy of ~2 decibels) can be achieved when selecting g_s approximately in the center of the possible dynamic range of the signals.

Now let us consider the problem of the adaptive Kalman filter in which $\tilde{\mu} = 10$ is selected for variation of the signal/noise ratio at the filter input within the limits of $\mu \varepsilon$ [3, 100].

Figure 5 shows the values of the steady state relative filtration error for three types of filters: the nonadaptive filter with fixed band (solid curves), the adaptive filter with iterative procedure with respect to $K_{\alpha\alpha_0}$ (the dotted curves), the adaptive filter with fixed $K_{\alpha\alpha_0} = 100$ (the dash-dot lines). From the figure the expediency of adaptation with respect to the unknown parameter α is obvious. Thus, for example, the filtration errors in the absence of adaptation reach ~7 decibels at the same time as for adaptation with respect to the parameter α and $K_{\alpha\alpha_0}$ they do not exceed ~2 decibels.

Thus, the proposed iterative procedure with respect to $K_{\alpha\alpha_0}$ in the adaptive Kalman filter is effective also for unknown spectral density of the signal

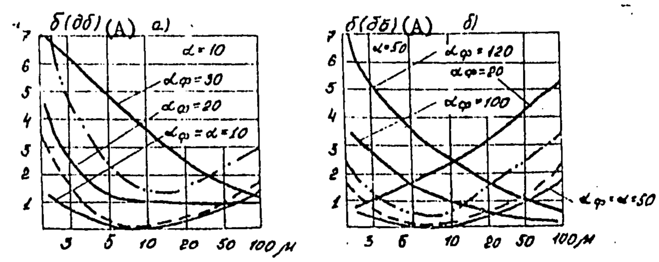

Figure 5.

Key: A. decibels

g_s. The advantage of this procedure is the fact that it is insensitive also to the initial value of the dispersion $K_{\alpha\alpha_0}$ for various values of the input signal band.

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UDC 621.391.2

PROCESS RECOGNITION ALGORITHM UNDER THE CONDITIONS OF A PRIORI INDETERMINACY

By N. G. Cherkashin, pp 138-139

Let us consider the problem of recognizing two a priori equiprobable process classes $Z(t)$ and $Y(t)$, $t \in T = [0, 1]$ having available training samples of volume n from both classes: $(Z_1(t), \ldots, Z_n(t)$ and $(Y_1(t), \ldots, Y_n(t))$. In order to find the solution it is necessary to construct the decision rule $\hat{\ell}$: "$\hat{\varepsilon}(X) \geq 0$ $(<0) \to X$ belongs to the first (second) class." We shall understand the processes as the limiting (with respect to dimensionality m) case of finite dimensional vectors obtained on breakdown of $T_m = \{t_j = j/m, j = 1, \ldots, m\}$ of the interval $T = [0, 1]$. Then it is possible to construct the optimal procedure in a defined sense for finite dimensional space, and then to proceed to the limit with respect to the dimensionality m.

The optimal decision rule in the sense of the minimum risk $\ell = p^{1/m} - q^{1/m}$ will be estimated, replacing the unknown densities by estimates, for example, nonparametric estimates. For the first class the density estimate has the form:

$$\hat{p}(x) = n^{-1}\sum_{i=1}^{n} 2^{-m}\left(\prod_{j=1}^{m} h(t_j)\right)\exp\left\{-\sum_{j=1}^{m} h(t_j)|X(t_j) - Z_i(t_j)|\right\},$$

for the second class, the analogous form. It is proposed that the "smoothing" parameters h be determined from the condition of minimum upper bound $\bar{R}(\hat{\ell})$ of the risk $R(\hat{\ell})$ from application of the rule $\hat{\ell} = \hat{p}^{1/m} - \hat{q}^{1/m}$:

$$R(\hat{\ell}) \leq \bar{R}(\hat{\ell}) = R(\ell) + \left(1 - \exp\left\{-\int p \log \frac{p}{\hat{p}}\, dx\right\}\right)^{1/2} + \left(1 - \exp\left\{-\int q \log \frac{q}{\hat{q}}\, dx\right\}\right)^{1/2},$$

where $R(\ell)$ is the Bayes (minimum) risk level. The last problem is equivalent to two problems for the maximum. Let us consider, for example, the first $h = \arg\max_h \int p(x) \log \hat{p}(x; h)\, dx$. The functional $J = \int p \log \hat{p}\, dx$ which is linear with respect to p is estimated as follows:

$$J = n^{-1}\sum_{i=1}^{n} \log \hat{p}(Z_i; h) = n^{-1}\log \prod_{i=1}^{n} \hat{p}(Z_i; h) = n^{-1}\log L(h; Z_1, \ldots, Z_n),$$

where

$$L(h)=L(h(t_1),\ldots,h(t_n))=\prod_{i=1}^{n}\sum_{\substack{s=1\\s\neq i}}^{n}\Big(\prod_{j=1}^{m}h(t_j)\Big)\exp\Big\{-\sum_{j=1}^{m}h(t_j)|Z_i(t_j)-Z_s(t_j)|\Big\}.$$

It is demonstrated that the parameters h reaching the maximum of the function L for finite m are calculated by the formula:

$$h_j=h(t_j)=\lim_{r\to\infty}\Big(\int_0^\infty\ldots\int_0^\infty h_j L^r(h_1,\ldots,h_m)dh_1\ldots dh_m\Big)\Big/\Big(\int_0^\infty\ldots\int_0^\infty L^r(h_1,\ldots,h_m)dh_1\ldots dh_m\Big),$$
$$j=1,\ldots,m.$$

and for $m\to\infty$ (considering that $\lim_{m\to\infty} t_j=\lim_{m\to\infty} j/m=t\in[a,1]$) have the form:

$$h(t)=\Big(n^{-1}\sum_{i=1}^{n}|Z_i(t)-Z_{\omega(i)}(t)|\Big)^{-1}, \tag{1}$$

where

$$\omega=(\omega(1),\ldots,\omega(n))=\arg\min_{\omega\in\Omega_n}\int_0^1\log\Big(n^{-1}\sum_{i=1}^{n}|Z_i(t)-Z_{\omega(i)}(t)|\Big)dt;$$

Ω is the set of $(n-1)^n$ vectors, the components of which assume values from the set $\{1, \ldots, n\}$ not equal to the number of the component, and for the rest arbitrary.

The limit (with respect to dimensionality m) of the decision rule $\hat{\ell}=\hat{p}^{1/m}-\hat{q}^{1/m}$ is equivalent in the sense of risk to the rule:

$$\hat{\ell}(X)=\max_{i=\{1,\ldots,n\}}\int_0^1(\log h_1(t)-h_1(t)|X(t)-Z_i(t)|)dt-$$
$$-\max_{i=\{1,\ldots,n\}}\int_0^1(\log h_2(t)-h_2(t)|X(t)-Y_i(t)|)dt,$$

where $h_1(t)$ is defined by formula (1), $h_2(t)$, by the same formula is we replace Q by Y in it.

Let us note that the limiting decision rule is similar to the known "nearest neighbor" rule. The convergence will be complete in the case where $h_1(t) = h_2(t) = h(t)$ and the closeness is understood in the sense of the norm $\|\cdot\|=\int_0^1|\cdot|h(t)dt$. Then the limiting decision rule assumes the form: $\hat{\ell}(X)=\min_{i=\{1,\ldots,n\}}\|X-Y_i\|_h-\min_{i=\{1,\ldots,n\}}\|X-Z_i\|_h$.

UDC 621.391

COMPENSATION METHODS OF SUPPRESSING LOCAL INTERFERENCE

By Yu. D. Bozhok, L. G. Krasnyy, pp 139-143

This paper is devoted to the investigation of the problems of the synthesis and analysis of optimal and quasioptimal channels to compensate for local interference and also estimate the effect of the a priori data on the effectiveness of these channels. Let us determine the structure of the optimal signal receiver $s(t-\vec{\alpha}_s\vec{x})$ in the field of the distributed and local interference. Let the local interference be formed by the set of plane waves arriving from the direction $\vec{\alpha}_i$ $(i = \overline{1, m})$, not coinciding with the direction $\vec{\alpha}_s$ of arrival of the signal. Then considering the narrow band signals, the optimal processing algorithm is described by the expression [1]:

$$u_0 = \left| \int_{\Omega} \int_T u(t,\vec{x}) B(t,\vec{x})\, dt\, d\vec{x} \right|, \tag{1}$$

where $U(t, \vec{x})$ is the adopted realization, $\vec{\Omega}$ is the region of space occupied by the receiving antenna, T is the signal duration, $B(t, \vec{x})$ is the solution of the equation:

$$\int_{\Omega}\int_T \left\{ R(t,t')\rho(\vec{x},\vec{x}') + \sum_{i=1}^{m} R_i(t,t') e^{j\omega_0 \vec{\alpha}_i(\vec{x}-\vec{x}')} \right\} B(t',\vec{x}')\, dt'\, d\vec{x}' = s(t) e^{j\omega_0 \vec{\alpha}_s \vec{x}}, \tag{2}$$

in which $R(t, t')$ and $R_i(t, t')$ are the complex time correlation functions of the distributed and ith localized interference respectively, $\rho(\vec{x}, \vec{x})$ is the spatial correlation function of the distributed interference on the central frequency ω_0 of the signal spectrum.

Lt us find the solution to equation (2) in the form:

$$B(t,\vec{x}) = B_s(t) L_s(\vec{x}) - \sum_{i=1}^{m} B_i(t) L_i(\vec{x}), \tag{3}$$

where $B_s(t)$ and $L(x)$ are the solutions to the following equations, respectively:

$$\int_T R(t,t')B_s(t')dt' = s(t), \tag{4}$$

$$\int_{\vec{\Omega}} \rho(\vec{x},\vec{x}')L(\vec{x}')d\vec{x}' = e^{j\omega_0 \vec{\alpha}\vec{x}}. \tag{5}$$

Then the unknown functions $B_i(t)$ satisfy the following system of integral equations:

$$\int_T R(t,t')B_i(t')dt' + \sum_{k=1}^{m} \Psi(\vec{\alpha}_k,\vec{\alpha}_i)\int_T R_i(t,t')B_k(t')dt' = \Psi(\vec{\alpha}_s,\vec{\alpha}_i), \tag{6}$$

where $= \int_T R_i(t,t')B_s(t')dt'$, $\quad \Psi(\vec{\alpha}_k,\vec{\alpha}_i) = \int_{\Omega} L_k(\vec{x})e^{-j\omega_0\vec{\alpha}_i\vec{x}}d\vec{x}$

is the spatial indeterminacy function of the radiation pattern of the optimal antenna.

Considering (3) the voltage (1) at the output of the optimal receiver:

$$u_0 = \left|\int_T B_s(t)\left\{\int_{\Omega} u(t,\vec{x})L_s(\vec{x})d\vec{x}\right\}dt - \sum_{i=1}^{m}\int_T B_i(t)\left\{\int_{\Omega} u(t,\vec{x})L_i(\vec{x})d\vec{x}\right\}dt\right| \tag{7}$$

From (7) it follows that the optimal receiver is multichannel: it contains a "signal" antenna oriented on the signal and "m" compensation antennas oriented in the directions of arrival of the local interference. The processing of the individual channels is separated into spatial and time processing where the phase-amplitude distributions $L(\vec{x})$ in the channels are optimized considering the spatial correlation function of the distributed interference (5), and the reference voltages $B(t)$, considering the time correlation functions of both the distributed and local interference (4), (6). After correlation processing, compensation is carried out (subtraction) of the local interference in the "signal" channel. In the special case $R_i(t, t') = \gamma_i R(t, t')$, where γ_i is the ratio of the dispersions of the ith local and distributed interference in the algorithm (7), $B_i(t) = \alpha_{si} B_s(t)$, where α_{si} is the solution of the system of algebraic equations:

$$\alpha_{si} + \gamma_i \sum_{k=1}^{m} \alpha_{sk}\Psi(\vec{\alpha}_k,\vec{\alpha}_i) = \gamma_i \Psi(\vec{\alpha}_s,\vec{\alpha}_i). \tag{8}$$

Correspondingly, the voltage at the output of the optimal receiver:

$$U_0 = \left|\int_T B_s(t)\left\{\int_{\Omega} u(t,\vec{x})L_s(\vec{x})d\vec{x} - \sum_{i=1}^{m}\alpha_{si}\int_{\Omega} u(t,\vec{x})L_i(\vec{x})d\vec{x}\right\}dt\right|. \tag{9}$$

In contrast to (7) in this algorithm only one time processing device is used (the correlator or matched filter), that is, all of the processing is broken down into spatial (with compensation of the local interference) and time.

Let us consider an example of the signal detection in the field of distributed and only local interference arriving from the direction $\vec{\alpha}_\ell$. In the signal band the spectrum of the interference in practice is planar; therefore $R(t,t') = g/2\,\delta(t-t')$. Let us also propose that the local interference is nonstationary with the correlation function $R_\ell(t,t') = g_\ell/2\, m^2(t)\,\delta(t-t')$. Substituting these expressions in (4) and (6) and also considering that for m = 1, $a_{\ell i} = \gamma_i \Psi(\vec{\alpha}_\ell, \vec{\alpha}_i)$, $a_{s\ell} = \gamma_l \Psi(\vec{\alpha}_s, \vec{\alpha}_l)$, instead of (7) we obtain:

$$u_0 = \frac{2}{g}\Big|\int_T s(t)\Big\{\int_\Omega u(t,\vec{x})L_s(\vec{x})d\vec{x} - \frac{g_\ell/g\,\Psi(\vec{\alpha}_s,\vec{\alpha}_\ell)m^2(t)}{1+g_\ell/g\,\Psi(\vec{\alpha}_\ell,\vec{\alpha}_\ell)m^2(t)}\int_\Omega u(t,\vec{x})L_\ell(\vec{x})d\vec{x}\Big\}dt\Big| \quad (10)$$

For strong local interference the algorithm is simplified:

$$u_0 = \frac{2}{g}\Big|\int_T s(t)\Big\{\int_\Omega u(t,\vec{x})L_s(\vec{x})d\vec{x} - \frac{\Psi(\vec{\alpha}_s,\vec{\alpha}_\ell)}{\Psi(\vec{\alpha}_\ell,\vec{\alpha}_\ell)}\int_\Omega u(t,\vec{x})L_\ell(\vec{x})d\vec{x}\Big\}dt\Big|.$$

Consequently, there is no necessity for special measures with respect to stationarization of the strong local interference -- stationarization is insured automatically during compensation. The result obtained has transparent physical meaning: local interference in the "signal" channel is read out with amplitude and phase normalized effect at the output of the compensation channel, and inasmuch as the normalization equalizes the local interference at the outputs of the two channels, leaving the nature of the modulating function identical, after readout, the nonstationary local interference is suppressed. Analogously, it is easy to prove that if the local interference is stationary and the distributed interference is nonstationary, in the "signal" and compensation channels, stationarization devices must be provided for which take into account the nature of the nonstationarity of the distributed interference.

The realization of the above synthesized optimal algorithms for suppressing local interference requires a priori data on the angles of arrival and the dispersions of the local interference (for determination of the coefficients α_{si}). The difficulties caused by the absence of information about the dispersion σ_i^2 of the local interference can be overcome as a result of modification of the algorithm (9):

$$u_0 = \Big|\int_T b_s(t)\Big\{\int_\Omega u(t,\vec{x})L_s(\vec{x})d\vec{x} - \sum_{i=1}^{m} C_{si}\int_\Omega u(t,\vec{x})L_i(\vec{x})d\vec{x}\Big\}dt\Big| \quad (11)$$

where the coefficients C_{si} are found from the system of equations:

$$\sum_{k=1}^{m} C_{sk}\Psi(\vec{\alpha}_k,\vec{\alpha}_i) = \Psi(\vec{\alpha}_s,\vec{\alpha}_i), \quad (12)$$

obtained by maximizing the signal/noise ratio with an additional condition of complete suppression of the local interference.

Analysis shows that the noiseproofness of the algorithm (11) in practice does not differ from the potential noiseproofness. In addition, for its realization a priori data on the disperse local interference are not required; however as before information is needed about their angles of arrival. Let us discuss this problem in more detail. The information about the angles of arrival $\vec{\alpha}_i$ of the local interference is used twice in the algorithm: first, to determine the coefficients C_{sk} in the system for joining the spatial channels, and secondly, for proper orientation of the radiation patterns of the compensation antennas. However, it turns out that the second factor is not so significant, that is, the orientation of the antennas of the compensation channels can be selected without considering the data on $\vec{\alpha}_i$. For proof of this statement let us calculate the signal/noise ratio α at the output of the spatial processing of the receiver (11) in which the static fan of the radiation pattern oriented in the direction $\vec{\beta}_i \neq \vec{\alpha}_i$ is used. If the radiation patterns are oriented so that $|\Psi(\vec{\beta}_i, \vec{\beta}_j)| = 0$, then in the presence of local interference alone

$$\alpha = (\Psi(\vec{\alpha}_s, \vec{\alpha}_s) |\tilde{\Psi}(\vec{\alpha}_1, \vec{\beta}_1)|^2) / (|\tilde{\Psi}(\vec{\alpha}_1, \vec{\beta}_1)|^2 + |\tilde{\Psi}(\vec{\alpha}_s, \vec{\alpha}_1)|^2), \tag{13}$$

where $\tilde{\Psi}(\vec{\alpha}, \vec{\beta}) = \Psi(\vec{\alpha}, \vec{\beta}) / \sqrt{\Psi(\vec{\alpha}, \vec{\alpha}) \Psi(\vec{\beta}, \vec{\beta})}$ is the normalized radiation pattern. a numerical comparison of this expression with the signal/noise ratio $\alpha_0 = \Psi(\vec{\alpha}_s, \vec{\alpha}_s)\{1 - |\tilde{\Psi}(\vec{\alpha}_s, \vec{\alpha}_1)|^2\}$ at the output of the spatial processing of receiver II with optimal orientation of the antennas demonstrates that $\alpha/\alpha_0 \approx 1$. It is easy to see that the validity of this expression also analytically if we approximate the main lobe of the radiation pattern $|\Psi(\vec{\alpha}, \vec{\beta})|$ by a function of the type: $\cos[\eta(\vec{\alpha}, \vec{\beta})]$. Thus, for example, for a linear antenna $|\tilde{\Psi}(\alpha_s, \alpha_1)| \approx \cos\{\frac{\pi}{2} \cdot \frac{L}{\lambda}(\sin\alpha_s - \sin\alpha_1)\}$, $|\tilde{\Psi}(\alpha_1, \beta_1)| \approx \cos\{\frac{\pi}{2} \cdot \frac{L}{\lambda_0}(\sin\alpha_1 - \sin\beta_1)\}$; therefore considering the orthogonality of the "signal" and compensation antennas we obtain: $\alpha/\alpha_0 \approx 1$. Consequently, there is no necessity for exact orientation of the compensation antennas of the local interference sources, and for suppression of the local interference it is possible to use the already formulated spatial channels. Let us note that for linear antennas this method of realizing the compensator is convenient also by the fact that on varying the angles of arrival of the local interference it is necessary to tune only the amplitudes of the weights c_{sk}, leaving their phases or delays unchanged, inasmuch as the latter depends only on the mutual arrangement of the radiation patterns of the spatial channels. Thus, the information about the angles of arrival of the local interference is in practice necessary only to select the weight coefficients at the outputs of the spatial channels. They can be obtained using the following adaptive procedure with training [2]: the strong local interference is detected and the direction found (using a precision direction finding channel) in the first step (for this interference is of the greatest danger); in the second step the estimates of the angles $\vec{\alpha}_i$ are used to determine c_{si}. During this processing the loss in the signal/noise ratio as a result of errors in measuring $\vec{\alpha}_i$ in the presence of local interference alone is:

$$\beta \approx 1+\gamma_1 \sigma_\alpha^2 |\Psi'(\vec{\alpha}_s)-\Psi'(\vec{\beta}_1)|^2 \Psi(\vec{\alpha}_1,\vec{\alpha}_1)/\Psi(\vec{\alpha}_s,\vec{\alpha}_s) |\Psi(\vec{\alpha}_s,\vec{\alpha}_s)|^2,$$

where $\Psi'(\vec{\alpha}_s)=\frac{\partial}{\partial\vec{\alpha}_1}\Psi(\vec{\alpha}_s,\vec{\alpha}_1)/\Psi(\vec{\alpha}_s,\vec{\alpha}_1)$ and $\Psi'(\vec{\beta})=\frac{\partial}{\partial\vec{\alpha}_1}\Psi(\vec{\beta},\vec{\alpha}_1)/\Psi(\vec{\beta},\vec{\alpha}_1)$ respectively, the steepness of the directional characteristic of the "signal" and compensation antennas in the direction $\vec{\alpha}_1$, the σ_α^2 dispersion of the estimate of the angle of arrival of the local interference. Inasmuch as $\alpha \approx \alpha_0$, and $\Psi'(\vec{\alpha}_s)$ and $\Psi'(\vec{\beta})$ have different values, $\beta \leq 1+\gamma_1\sigma_\alpha^2/\Psi'(\alpha_s)/^2$. If in addition it is considered that the minimum dispersion of the bearing estimate $\sigma_\alpha^2 \approx \{-\gamma_1 \Delta f T_u \frac{\partial^2}{\partial\vec{\alpha}^2}|\Psi(\vec{\alpha}_s,\vec{\alpha}_1)|^2_{\vec{\alpha}_1=\vec{\alpha}_s}\}^{-1}$ where T_u is the bearing measurement time, Δf is the preselector band of the receiving channel, then we obtain:

$$\beta \leq 1+(|\Psi'(\vec{\alpha}_s)|^2)/(\Delta f T_u \frac{\partial^2}{\partial\vec{\alpha}^2}|\Psi(\vec{\beta}_1,\vec{\alpha}_1)|^2_{\vec{\alpha}_1=\vec{\beta}_1}).$$

Thus, the loss with respect to noiseproofness of the compensation channel as a result of the error in measuring $\vec{\alpha}_i$ does not depend on the intensity of the local interference. In the special case of a linear antenna

$$\beta \leq 1+\frac{12}{\pi^4 \Delta f T_u}.$$

Consequently, the adaptive channel with preliminary measurement of the bearing permits effective compensation of the local interference.

In addition to the described adaptive method of compensation with preliminary estimation of $\vec{\alpha}_i$, another method of synthesizing the compensator is possible based on direct calculation of the weight coefficients. Its essence consists in the following. Based on the results obtained above it is proposed that for compensation antennas are used which are oriented in the direction of $\vec{\beta}_i \neq \vec{\alpha}_i$ where $|\Psi(\vec{\beta}_i,\vec{\beta}_j)| = 0$. Then instead of (8) we obtain

$$\sum_{i=1}^{m} a_{si}[\Psi(\vec{\beta}_i,\vec{\beta}_j)+\sum_{\ell=1}^{m}\gamma_\ell\Psi(\vec{\beta}_i,\vec{\alpha}_\ell)\Psi^*(\vec{\beta}_k,\vec{\alpha}_\ell)]=\sum_{\ell=1}^{m}\gamma_\ell\Psi(\vec{\alpha}_s,\vec{\alpha}_\ell)\Psi^*(\vec{\beta}_k,\vec{\alpha}_\ell). \qquad (13)$$

It is easy to show that the function $K_{ki}=\Psi(\vec{\beta}_i,\vec{\beta}_k)+\sum_{\ell=1}^{m}\gamma_\ell\Psi(\vec{\beta}_i,\vec{\alpha}_\ell)\Psi^*(\vec{\beta}_k,\vec{\alpha}_\ell)$ is equal with accuracy to the constant to the mutual correlation function of the interference at the outputs of the kth and ith spatial channels, that is, $K_{ki}=E\{u_k^*(t)u_i(t)\}$, where $u_i(t)=\int_\Omega u(t,\vec{x})L_i(\vec{x})d\vec{x}$. Consequently,

$$\sum_{i=1}^{m} K_{ki}a_{si}=K_{ks}-\Psi(\vec{\alpha}_s,\vec{\beta}_k). \qquad (14)$$

Applying the method of stochastic approximations to this equation, we obtain the recurrent relations for calculating the coefficients a_{si}:

$$a_{sk}^{(n)} = a_{sk}^{(n-1)} - \mu_n \{u_k^*(t_n)[u_s(t_n) - \sum_{i=1}^{m} a_{si}^{(n-1)} u_i(t_n)] - \psi'(\vec{\alpha}_s, \vec{\beta}_k)\}, \tag{15}$$

where μ_n are the weight coefficients insuring convergence of the procedure (14).

The modification of the algorithm (14) based on the smooth estimates of the correlation functions K_{ki} is also possible:

$$a_{sk}^{(n)} = a_{sk}^{(n-1)} - \frac{1}{\mu_n}\{\frac{1}{n}\sum_{z=1}^{n} u_k^*(t_z)[u_s(t_z) - \sum_{i=1}^{m} a_{si}^{(z-1)} u_i(t_z)] - \psi'(\vec{\alpha}_s, \vec{\beta}_k)\}. \tag{16}$$

The advantage of the algorithms obtained is their invariance with respect to the a priori information about the angles of arrival and the dispersions of the local interference.

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DETECTION CHARACTERISTICS OF A SIGNAL WITH UNKNOWN ANGLE OF ARRIVAL

By V. K. Marshakov, Yu. S. Radchenko, A. P. Trifonov, pp 144-145

Let the transmitting antenna with circular radiation pattern emit a rectangular radio pulse of duration T. This pulse, being reflected from a point target with unknown azimuth ϕ_0 reaches the input of the receiving antenna with intrapulse beam scanning. Considering that the radiation pattern of the receiving antenna azimuthally has a bell-shape $\exp[-(\phi - \phi)^2/\gamma^2]$ and its scanning is carried out by a linear law with frequency Ω in the range of angles $[-\phi_m/2; \phi_m/2]$, the signal at the output of the antenna will be written in the form [1]: $x(t)=s(t,\tau_0)+n(t)$. Here $s(t,\tau_0)=A\exp[-(t-\tau_0)^2\Omega^2/\gamma^2]\cos(\omega_0 t+\psi_0)$ is the useful signal $t_0 = \phi_0/\Omega$, n(t) is a realization of stationary normal noise with spectral density:

$$N(\omega)=\left[4\sigma_r^2\frac{\sin^2(\omega-\omega_0)T/2}{(\omega-\omega_0)^2T}+N_s\right]\gamma\sqrt{\frac{\pi}{2}}+N_0,$$

σ_r^2 is the dispersion of the reverberation interference, N_3 and N_0 are the spectral densities of the sea noise and the natural noise of the receiving system.

One of the most widespread detection devices is a receiver made up of a matched filter, a linear detector of the envelope and a threshold device. The detection of the signal with unknown angle of arrival in such a receiver is by comparison with the given threshold H of the absolute maximum of the process at the output of the envelope detector in the time interval $[-\varphi_m/2\Omega; \varphi_m/2\Omega]$ Thus, the detection algorithm can be written as follows:

$$\max_\tau M(\tau)=\max_\tau \left|\int_0^T x(t)s(t,\tau)dt\right| \gtrless H. \tag{1}$$

According to (1) in order to determine the detection characteristics it is necessary to find the distribution functions of the absolute maxima $M(\tau)$ in the presence and absence of a signal. This problem has no exact solution. However, by using the proposition of the Poisson nature of the blips of the stationary random process for a sufficiently high level and the results of reference [2], it is possible to find asymptotically exact expressions for the detection characteristics. Thus, for the probability of false alarm it is easy to obtain

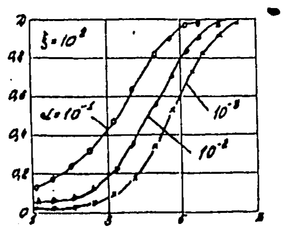
$$\alpha = \begin{cases} 1 - \exp\left[-\frac{\xi^*}{\sqrt{2\pi}} \frac{H}{\sigma} \exp\left(-\frac{H}{2\sigma^2}\right)\right]; & H/\sigma \gg 1; \\ 1 & H/\sigma < 1. \end{cases} \quad (2)$$

Here $\sigma^2 = z^2\{1 + z_1^2[\sqrt{2\pi} - 2(1-\exp(-\xi^2/2))/\xi]\}$, $\xi^* = \xi \frac{z}{\sigma}\sqrt{1 + z_1^2[1-\exp(-\xi^2/2)]/\xi}$, $\xi = \varphi_m/\gamma$, $z^2 = A^2/\Omega[(N_0/\gamma\sqrt{\frac{\pi}{2}}) + N_s]$, $z_1^2 = \sigma_r^2/[(N_0/\gamma\sqrt{\frac{\pi}{2}}) + N_s]$. The probability of missing a useful signal in this case is determined by the expression:

$$\beta = (1-\alpha)\left[1 - Q\left(\frac{z^2}{\sigma}, \frac{H}{\sigma}\right)\right], \quad (3)$$

where Q(x, y) is the Markum function.

It is necessary to note that expressions (2) and (3) for finite values of the signal/noise ratio Z and the number of resolution elements azimuthally are approximate, and they become exact only with an unlimited increase in Z and ξ.



In order to establish the limits of applicability of the presented formulas (2), (3) for finite values of z and ξ, physical simulation of the process of detecting a signal with unknown angle of arrival was carried out. In the figure we have the experimental values obtained for the probability of correct detection by the Neuman-Pearson criterion for various values of z and the probability of false alarm α. In the same figure the solid lines are used to plot the theoretical functions calculated by formulas (2) and (3). As follows from a comparison of the theoretical and experimental data, the asymptotically exact relations obtained satisfactorily approximate the experimental results for $z > 1$, $\xi \geq 10$.

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QUASIOPTIMAL DETECTION OF MULTIBEAM SIGNALS IN THE PRESENCE OF A PRIORI UNKNOWN FLUCTUATION RATE OF THE PARAMETERS OF THE PROPAGATION CHANNEL

By A. G. Golubev, pp 145-147

In [1] a study was made of the algorithms for detecting multibeam signals -- SEA (sliding empirical algorithms) -- under the assumption that the channel parameters are stable during some a priori known time interval τ_k equal to the duration of the element signal of the initial (sounding) bunch, after which they vary discontinously. In practice the value of τ_k, as a rule, is a priori unknown. Let us consider the SEA constructed analogously [1], but calculated for the correlation interval of the fluctuations of the channel parameters τ_{min} which is less than the actual τ_k (it is assumed that the exact upper bound of the possible fluctuation rate of the channel parameters is a priori known).

The statement of the detection problem is analogous to [1] except that in the given case in the initial bunch there are N_k signals, where $k = \tau_k/\tau_{min}$ is the safety margin about which it is a priori known that $k \geq 1$; the values of the amplitudes $H_{i\ell}$ and H_{ij} (the phases $\phi_{i\ell}$ and ϕ_{ij}) of the ith beam at the propagation times of the ℓth and jth signals of the bunch respectively (ℓ, j = 1 to N_k) fluctuate together for $|j - \ell| > k$ to independently for $|j - \ell| \geq K$. The following methods of organizing the SEA are analyzed (the principles of breaking down the bunch into two series: the training series (OP) and the working series (RP)):

The SEA-1 direct method: all the signals of the bunch from the 1st to the $(N_k/2)$-st are the training sequence, and the signals from $(N_{k+1}/2)$th to the nk-th are the working sequence;

The method with mixing SEA-1: all the odd signals in the bunch are training, and the evens, working;

SEA-3: all the signals of the bunch are used both as training and as working sequences. Figure 1 shows the block diagram of the SEA-1 constructed by the method with "mixing." For execution of the SEA-3 it is necessary to find the vector sums of the outputs of the delay line elements in a common adder, after which it is necessary to calculate the square of the norm of the result obtained. It is demonstrated that most effective is separation of the signals in the bunch fluctuating together into different subsequences, and the signals fluctuating independently, into like sequences (both signals in the training sequence or both in the working sequence). Hence, it follows that the "method with mixing" is more effective than "the directmethod." An analysis was made

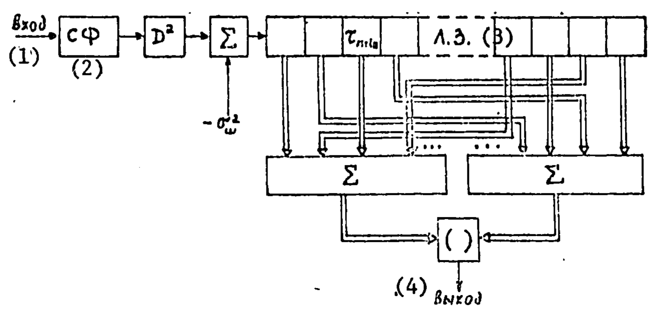

Figure 1. SF-filter, matched with one signal of the initial bunch (periodic bunch); D^2 -- quadratic detector; LZ -- delay line containing N_k elements, each of which insures a delay of τ_{min}; ( ) -- the block for calculating the scalar product; Σ -- the adder; the double arrows indicate the vector relations.

Key:
1. input
2. SF filter
3. LZ delay line
4. output

of the noiseproofness of the algorithms in the gaussian approximation of the distribution densities of the processes at the outputs of the adders of processing the training and operating sequences and the process at the output of the SEA. On the basis of the calculations performed for a broad class of situations, it is possible to draw the following conclusions. The "method with mixing" SEA-1, just as was proposed, insures the required detection characteristics at lower (by 20 to 35%) powers of the threshold signals than the "direct method" SEA-1. The most effective of the investigated algorithms is SEA-3, but for even k, the "method with mixing" SEA-1 loses in power of the threshold signal to the algorithm SEA-3 a total of 15 to 20%. The last result permits calculation that for even k the potential possibilities can be realized when using the "method with mixing" SEA-1 constructed in the form of a multistep algorithm, that is, an algorithm with successive multiple division of the training and working sequences into two sequences.

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LINEAR DISCRETE PROCESSING WITH QUADRATIC RESTRICTIONS OF THE SIGNAL DISTORTIONS IN AN ADAPTIVE SYSTEM

By Yu.P. Podgayskiy, pp 147-149

The application of a adaptive spatial filters at the output of an acoustic antenna for linear processing of signals is based on the synthesis of the filter structure by adaptation of its parameters. In order to fix the sensitivity of the system in the observation direction, a selective linear restriction is introduced on the variation of the parameters of the spatial filter [1]. It is possible to increase the stability of the adaptive systems with respect to errors consisting in deviation of the filter parameters from the established limits, as the result of an auxiliary operation from which the system sensitivity to the errors increases as the square of the deviation of the current value of the weighted vector of the filter from the value insuring constancy of the transmission coefficient in the direction of the observation. Let it be necessary to isolate the acoustic signal from a mixture of it with additive anisotropic acoustic noise. The spatial filter "remembers" J sample values of the processes on k elements of the antenna which are successive in time, forming KJ inputs of the filter. Let W be a column vector of the scalar weight coefficient ω_i, $i = 1, \ldots, KJ$ uniqely defining the filter. The set of weights ω_i of the filter minimizes the means square error T(W) of the signal estimate at the output of the system with restrictions, forming the vector of the type:

$$W = R_x^{-1} c (c^T R_x^{-1} c)^{-1} G. \quad (1)$$

Here (^{T}) and ($^{-1}$) are the signs of transposition and inversion of matrices. The vector G of dimensionality J and the block conversion matrix c = diag $[c_1, \ldots, c_j, \ldots, c_J)$ where the blocks $c_j = [c_{j1}, \ldots c_{j\ell}, \ldots, c_{jk}]$ have the same sense as in reference [1], R_x is the correlation matrix (the KJ-dimensional vector x of the input process) which is unknown to the system.

For the investigated case the value of T(W) appears in the form: $T(W) = \hat{\varepsilon}^2 + \beta^2 (C^T W - \hat{G})^T (C^T W - G)$ where ε^2 is the mean square error without restriction, the value of β^2 is selected from the positions of synthesis of the vector W_0 (see expression (1)): $\beta^2 >> (C^T R_x^{-1} C)^{-1}$. For $W = W_0$ the value of $T(W) = T_{min}$.

The use of the method of stochastic approximation [2] for synthesis of the filter during the process of its functioning will permit us to get away without the calculations (KJ × KJ) of the matrix elements R_x and R_x^{-1}. The systems are moved in the space of the errors T(W) in accordance with the algorithm:

$$W(n+1) = W'(n+1) + \mu\beta^2 C[G - C^T W(n)], \tag{2}$$

where W'(n + 1) is the algorithm giving the unconditional minimum ε^2_{min} [3]. The second term is not necessarily equal to zero. It prevents accumulation of the control errors for continuous operation of the system. On the basis of the random nature of the weight vector W(n), as the functions of the random vector X(n), the vector W(n) approaches on the average to a value of (1):

$$\lim_{n\to\infty} \langle W(n) \rangle = W_0 .$$

It is possible to show that for linear restrictions the algorithm is more complex than algorithm (2), and it is represented by the expression: $W(n = 1) = PW'(n + 1) + C(C^T C)^{-1} G$, where $P = I - C(C^T C)^{-1} C^T$ is the known matrix, I is the unit matrix of rank KJ. Let us note that for linear and quadratic restrictions the optimal vectors W_0 just as should be expected, coincide.

Thus, by introducing restrictions into the signal processing the linear distortions of its frequency spectrum can be reduced to a minimum by using algorithm (2) and selecting the transmission coefficient established by the vector.

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RECURSIVE ANALYSIS AND SYNTHESIS OF SIGNALS IN STATISTICAL HYDROACOUSTICS

By B. M. Kurilov, pp 149-151

In this paper it is proposed that the method of recursive modulation analysis be used for parametric representation of nonstationary hydroacoustic signals [1]. The essence of this method consists in the following. Let $\{\Phi(x)\}$ be the set of functions given in a finite interval $[x_0, x_m]$ such that their Fourier series has a finite number of terms, that is, $f(x)=\sum_{n=-n_{max}}^{n_{max}} C_n e^{jn\omega_0 x}$, $n_{max} < \infty$, where $C_n=\frac{1}{T}\int_0^T f(x)e^{-j\omega_0 nx}dx$ are the Fourier coefficients, $\omega_0 = 2\pi/T$, $T = x_M - x_0$. Then functions of this class the following terms are valid. Theorem 1. For any function $f(x)$ $\{\Phi(x)\}$ there is a finite value such that it is uniquely represented by the parametric series

$$f(x)=\frac{1}{N}\sum_{m=1}^{N} f(x_m)(\sin\{(\omega_c+\frac{\omega_0}{2})(x-x_m)\})/(\sin\{\frac{\omega_0}{2}(x-x_m)\}), \tag{1}$$

the coefficients of which are the reckonings of the values of the functions uniformly arranged in the interval T with the stepsize $(T/2)(n_{max} + 1)$, $\omega_c = \omega_0 n_{max}$. Let us consider a class of functions belonging to $\{\Phi(x)\}$ which are multiextremal. Let us represent $f(x)$ according to theorem 1 in terms of its discrete reckonings $f(x_m)$, and among them let us find all the local extrema. Let us break down the function in the interval of assignment $[x_0, x_M]$ into segments lying between adjacent extremal values, and let us represent each such section in the form of a discrete function so that:

$$K_i(x_m) = \begin{cases} f(x_m) & \text{for } x_m \in [x_i, x_{i+1}]; \\ 0 & \text{for } x_m \notin [x_i, x_{i+1}], \end{cases}$$

then

$$f(x_m)=K_i(x_m)\ ;\ x_m \in [x_i, x_{i+1}]\ ;\ i=1,2,\ldots,L;$$

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where x_i is the value of the argument for which the discrete function $f(x_m)$ has local extrema $f(x_i)$, L is the number of local extrema. Let us introduce the functions of a continuous argument $K_i(x)$ defined in terms of the discrete functions $K_i(x_m)$ as follows: the interval $[x_i, x_{i+1}]$ will be taken as half the quasiperiod $T_i/2$ of the discrete function $K_i'(x_m)$ and it will be supplemented symmetrically to the quasiperiod T_i. Let us reproduce the function of a continuous argument $K_i(x)$ in the quasiperiod using the parametric series (1), that is,

$$K_i(x) = 1/N_i \sum_{m=m_i}^{m_i+N_i} K_i'(x_m)(\sin\{(\omega_{ci} + \tfrac{\omega_{0i}}{2})(x - x_m)\})/(\sin\{\tfrac{\omega_{0i}}{2}(x - x_m)\}),$$

where N_i is twice the number of discrete points of the function in the interval from x_i to x_{i+1}, ω_{ci} is the lower boundary frequency, $(x_{i+1} - x_i)$ is the distance between adjacent extremal points, $\omega_{ci} = \omega_{0i} n_{imax}$ is the upper limiting frequency, $2(x_{i+1} - x_i)$ is the quasiperiod. Let us consider the case where $f(x)$ is such that for each function $K_i(x)$ the inequality $\omega_{ci} < 2\omega_{0i}$ is satisfied. We shall call the functions which satisfy this condition quasi-octave, and the class of such functions will be denoted as $\{\Phi(x)\}$.

Theorem 2. For any function $f(x) \in \{\Phi_0(x)\}$ and the presented parametric series (1), the coefficients of the series $f(x_m)$ are uniquely defined by the values of the function at the extremal points $f(x_i)$, the spacing between them $\tau(x_i)$ and the constant λ by the expression:

$$f(x_m) = f(x_i) + f(x_{i+1}) + |f(x_i) - f(x_{i+1})| (-1)^i \lambda \cos\{\tfrac{\pi}{\tau(x_i)}(x_m - x_i)\},$$

where $\tau(x_i) = x_{i+1} - x_i$; $x_m \in [x_i, x_{i+1}]$; $i = 1, 2, \ldots, L$

Theorem 3. For any $f(x) \in \{\Phi_0(x)\}$ there are the modulating functions $\tilde{A}(x)$, $\tilde{\rho}(x)$ and the aligning function $R(x)$ in $\{X\}$ belonging to the class $\{\Phi(x)\}$ and having the discrete finite spectrum no higher than $(\omega_c - \omega_0)/2$, which:

$$f(x) = \frac{1}{N} \sum_{m=1}^{N} \{[a + \tilde{A}(x_i)] \cos\{[\rho + \tilde{\rho}(x_i)](x_m - x_i)\} (-1)^i \lambda + R(x_i)\} \frac{\sin\{(\omega_c + \frac{\omega_0}{2})(x - x_m)\}}{\sin\{\omega_{0/2}(x - x_m)\}}, \quad (2)$$

where a, ρ and λ are constant;

Let us then consider the class of functions $f(x) \in \{\Phi_0(x)\}$ such that if they are represented in the form of expression (2), the variable components of their modulating functions $\tilde{A}(x)$ and $\tilde{\rho}(x)$ also belong to the class $\{\Phi_0(x)\}$. We shall state that such functions belong to the class Φ_1 and denotes this as $f(x) \in \{\Phi_1(x)\}$. Then the variable components of the modulating functions of the first order $\tilde{A}(x)$ and $\tilde{\rho}(x)$ can be, in turn, represented in terms of the modulating functions and the second-order constant components by the expressions analogous to (2). Now if the variable components of the second-order modulating functions also belong to the class $\{\Phi_0(x)\}$, we shall state that $f(x) \in \{\Phi_2(x)\}$, and so on. If all of the variable components of the modulating functions to the ℓth order belong to the class $\{\Phi_0(x)\}$, then $f(x)$ $\{\Phi_\ell(x)\}$.

Theorem 4. Any function $f(x) \in \{\Phi_\ell(x)\}$ is uniquely represented by the ℓth order finite matrices $\| d_{\ell\alpha} \|$; $\| \rho_{\ell\alpha} \|$; $\| \lambda_{\ell\alpha} \|$, the elements of which are constant components in the recursive expansion $f(x)$ with respect to its modulating functions, and the modulating functions on the order of $\ell \geq \log_2 N$ are identically equal to zero. However, the actual hydroacoustic signal can not belong to the class $\{\Phi_\ell(x)\}$. In this case it is proposed that we proceed as follows: for the initial hydroacoustic signal determine the extremal values and the distances between them and represented in the form of the sum of the quasiactive function $\Phi_1(x) \in \{\Phi_0(x)\}$ synthesized with respect to (2) and some remainder $Q_1(x)$ which is the difference between the initial nonquasioctave function and the quasioctave function $\Phi_1(x)$ synthesized with respect to (2), that is, $f(x) = \Phi_1(x) + Q_1(x)$ or

$$f(x_m) = R_1(x_l) + [\alpha_1 + \tilde{A}_1(x_i)](-1)^l \lambda_1 \cos\{[\rho_1 + \tilde{\rho}_1(x_i)](x_m - x_i)\} + \\ + Q_1(x_m); \quad x_m \in [x_i, x_{i+1}]; \; l = 1, 2, \ldots, L. \tag{3}$$

The difference function $Q_1(x_m)$ can again be represented by the expression (3): $Q_1(x_m) = \Phi_2(x_m) + Q_2(x_m)$, and so on. In the kth step of this series expansion, the function $f(x_m)$ will be represented by the parametric series with the remainder $Q_k(x_m)$,

$$f(x_m) = \sum_{K=1}^{K} \{[\alpha_K + \tilde{A}_K(x_i)] \cos\{[\rho_K + \tilde{\rho}_K(x_i)](x_m - x_i)(-1)^L \lambda_K + R_K(x_i)\} + Q_K(x_m). \tag{4}$$

Let us investigate the special case where $f(x) \in \{\Phi(x)\}$ is such that all the variable components of the modulating functions $\tilde{A}_k(x_i)$ and $\tilde{\rho}_k(x_i)$ in the expansions (4) is identically equal to zero. In this case all the functions $R_k(x_i)$ will also be equal to zero and (4) can be written in the form:

$$f(x_m) = \sum_{K=1}^{n_{max}} \alpha_K \sin(K \rho_1 x_m) \lambda_K \; .$$

The expression obtained essentially gives an expansion of the odd discrete function $f(x_m)$ in the harmonic Fourier series. A comparison of the expression obtained with (4) offers the possibility of drawing an important conclusion that the recursive modulation analysis of the functions from the class $\{\Phi(x)\}$ gives the same result as the Fourier analysis in the case where these functions are stationary, that is, their spectral components are constant in the analysis interval; if the functions are nonstationary and, consequently, $\tilde{A}_k(x)$ and $\tilde{\rho}_k(x)$ are not equal to zero, the Fourier analysis gives only values of the spectral components not reflecting their variations and, in this sense, is not complete. Considering the fact that the actual hydroacoustic signal is nonstationary, it is possible to hope that the recursive modulation analysis of this signal will give a more complete and adequate parametric description than the traditional Fourier analysis for many applied problems of statistical hydroacoustics.

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UDC 534.771

SOME POSSIBILITIES OF USING HEARING MODELS WHEN PROCESSING HYDROACOUSTIC INFORMATION -- ABSTRACT OF THE REPORT

By V. P. Ryzhov, Ye. A. Semik, M. N. Surkov, V. M. Chernyshev, pp 151-153

In this report an effort is made to generalize the results of the papers (a bibliography of 12 entries) on processing of hydroacoustic information by an operator, and a study is made of the possibilities of improving the effectiveness of processing of the information by the operator. It is noted that as applied to hearing models the decision making processes are more frequently depicted by abstract logical mathematical models, and the primary processing, by physical models.

The advantages of the operator over the automatic systems are manifested first of all, when solving the problems of signal recognition, constructing models of the objects of investigation and forecasting. However, even in such simple problems as detection, the estimation of one parameter, the operator has a number of advantages over automatic systems caused by the possibilities of adaptation to variations of the signal and interference characteristics, the large dynamic and frequency range, the high resolution with respect to frequency, and so on. It is necessary to consider the joint estimation of the statistical, semantic and valuable aspects of processed information a special property of the human operator. The use of the signal operator of the type of LFM pulses permits, just as in automatic systems, an increase in the noiseproofness with respect to the reverberation interferences. In the table experimental values are presented of the threshold signal/noise ratios in the form of voice-frequency and LFM pulses with different frequency deviation and also pulses with filling in the form of a meander with signal reception against a background of wide band noise and reverberation interference simulated in accordance with the canonical model. The duration of all of the signals was 200 milliseconds; the central frequency was 1 kilohertz. The data were obtained by averaging with respect to the reception results by several operators. The relative scattering of the threshold values of different operators did not exceed 20%. The threshold signal was defined with a probability of false alarm of 10^{-4} and a probability of correct detection ρ.

Table 1

Signal		Voice-frequency	Meander	LFM Δf = 1 kilohertz	LFM Δf = 260 hertz
Wide band	ρ	0.75	0.83	0.78	0.88
Noise f = 20 kh	$q = A/\sigma$	0.15	0.133	0.163	0.16
	$Q = (2E/S_0)^{1/2}$	9.5	8.4	10.4	9.75
Reverberations	ρ			0.95	0.85
	q			0.61	1
	Q			8.7	7.2

As follows from the experimental data, the threshold signal for voice-frequency and LFM pulses applied against a background of wideband interference is in practice identical at the same time as for the case of reception against a background of reverberation the threshold value for the LFM pulses increases. The decrease in the threshold signal on detection of pulses with meander filling is obviously connected with the fact that the harmonics fall in different frequency groups. The most important signs clearly recognized by the operator are the format structures and the basic tone of the quasiperiodic signals. In order for these signs to appear, the lower frequency of the spectrum (the frequency of the basic tone) must be reduced to 200-500 hertz. The hypothesis advanced in musical psychoacoustic about the special clarity of perception of short formants [1], in particular, the voice of a singer with clearly expressed short formants in the 2500 to 3000 hertz range has noiseproofness 20 decibels higher than the voice of a singer without such formants is of interest on the level of distinguishing signals.

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PECULIARITIES OF THE STRUCTURE OF THE CHANNEL FOR SPATIAL PROCESSING OF ACOUSTIC SIGNALS IN VERTEBRATES

By S. V. Pasechnyy, B. V. Solukha, A. B. Chubukov, pp 154-158

In this paper a discussion is presented of some of the peculiarities of the construction of the channel for spatial processing of acoustic signals in animals for binaural direction finding of sources. The directivity characteristic of the auditory system is estimated by the dependence of the probability of correct reactions of the animal to stimulus on the angle of irradiation with a fixed value of false reactions. Special attention has been given to the investigation of the directional characteristic of a binaural channel on the level of joint processing of signals coming from each sensory channel. A model of a binaural channel [1] (Figure 1) was used as the basis for investigating the possible methods of signal processing, in the structure of which anatomical, electrophysiological and psychoacoustic data were used. The vibrations of the eardrum are mechanically transmitted through the system of the middle ear with the transmission coefficient K and frequency filtration Φ_0 to the inner ear. The neuron structures form multilevel matrices with numerous intra- and inter-matrix couplings and projections in the higher divisions of the auditory system. In vertebrates the interaction between the binaural pulse fluxes, as a rule, begins with the superior olive level (blocks 5, 6 and 7). Some of the cells of the nuclei of the superior olive react differently depending on which ear receives the stimulus first. Both the lower and upper centers participate in the time and space processing, for the experiments indicate the impossibility of instantaneous determination of the direction of the sound source without participation of the auditory cortex (blocks 8, 9). The study of the spatial processing in such a system must be performed on the basis of measuring the directional characteristic of the system as a whole by behavioristic methods, and the studies of the role of the individual structure in the formation of the directional characteristic must be performed by electrophysiological methods. One of the statistical criteria for decision making about the presence of reactions of the system to the signal, for example, the Neuman-Pearson criterion, must be used in both situations.

As the performed experiments demonstrated, on stimulation frequencies to 10 kilohertz the monoaural directional characteristic of the auditory channel of a lake frog is close to circular. This fact is easily explained by the small size of the body structures of the animal by comparison with the wave length of the oscillations. Thus, binaural processing basically is determined by the neuron structures. The data from electrophysiological experiments indicate that the position of the animal under free field conditions essentially influences the threshold characteristics of the induced potential of the mesencephalon of the lake frog in response to audible clicks [2]. The results of the electrophysiological studies of different regions of the auditory analyzer belonging to the mesencephalon demonstrated a clearly expressed relation of the nuclei of the mesencephalon to the cochlea of the inner ear contralateral with respect to it. The quantitative characteristics obtained demonstrated the essential dependence of the form of the binaural directional characteristic on the localization of the electrode in the nuclei. The differences in thresholds of leads from the vicinity of the laminar and comissural nuclei of the torus were 6-8 decibels with identical position with respect to the radiator and a constant level of stimulation. The functioning of such mechanisms having the possibility of pattern scanning can have a significant effect on the noiseproofness of the system and lead to realization of the function detected in experiments on man [3].

Measurements were made of the directional characteristic of the auditory system on appearance of quivering fright in the animals on stimulation by powerful clacking sounds. The detection of the quivering reaction in the animal was made just as in the electrophysiological experiments. In practice all of the blocks of the receiving part of the system and one of the effector mechanisms determining the quivering reaction (Figures 1, 12) participate in the processing of the signals under such experimental conditions.

During the process of the experiment, the probability of the response of the animal to a stimulus by a quivering reaction was measured with respect to the number of times the threshold was exceeded by the signal from the transmitter one second after appearance of the stimulus with fixed probability of false reaction. The animal was stimulated by radio pulses 60 milliseconds long with smoothed fronts and different filling frequencies in the range of 100 to 15,000 hertz.

As a result, ellipsoidal directional characteristics were obtained (see Figure 4). Increasing the measurement threshold in both cases led to a change in shape of the direction of the characteristic which forces the proposition of the presence of nonlinear elements in the processing system. At higher thresholds the directional characteristic differs significantly from circular. For stimulation frequencies of 0.5 to 1 kilohertz this relation is manifested more clearly than for higher frequencies. Consequently, the formation of the directional characteristics obtained is realized exclusively as a result of functioning of the neuron structures.

It is necessary to note that in accordance with the experimental procedure, the directional characteristics obtained indicate the direction of arrival of the signal causing the most clearly expressed quivering reaction . In this situation the animal does not himself try to increase the noiseproofness of the system and form a narrower pattern. Therefore the directional characteristic obtained has ellipsoidal shape and is essentially broader than those detected in the electrophysiological experiment. A still narrower directional characteristic can be expected when measuring the search reactions or orientation reactions.

As a result of the performed experiments it is possible to conclude the following: bioacoustic procedures based on using classical threshold criteria have been developed which will permit investigation of the structure of the binaural channel and estimation of its operating principles from the probability point of view;

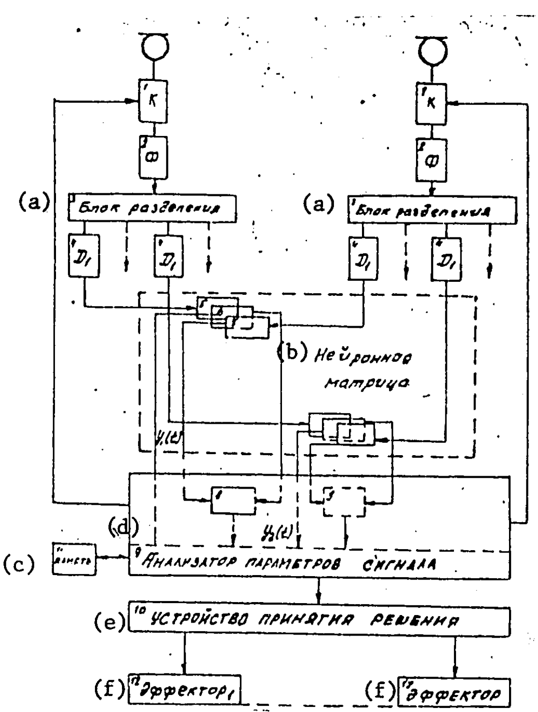

Figure 1. Structural diagram of the auditory system

Key:
a. separation block
b. neuron matrix
c. memory
d. signal parameter analyzer
e. decision making unit
f. effector

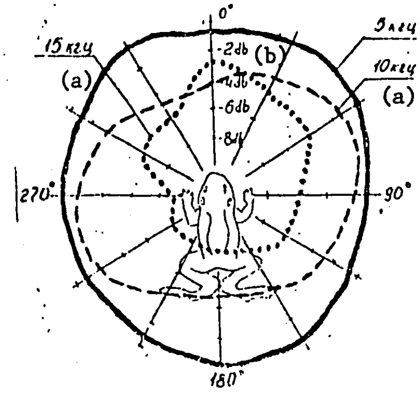

Figure 2. Monoaural radiation patterns of the auditory analyzer for tactile and remote stimulation.

Key: a. kilohertz
b. decibels

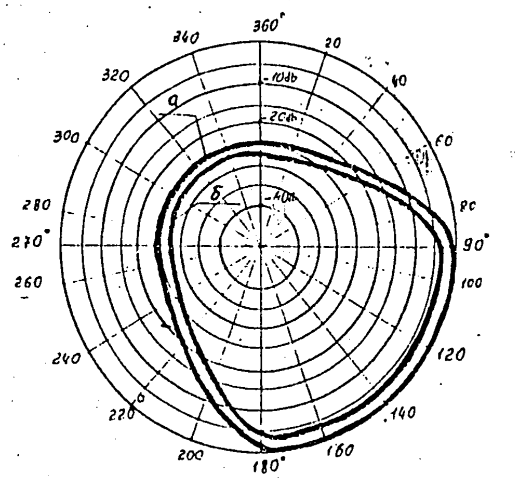

Figure 3. Radiation pattern of auditory analyzer when tapping the induced potential of the mesenchephalon. a -- threshold with a decrease in the sound pressure level; b -- threshold with an increase in the sound pressure level.

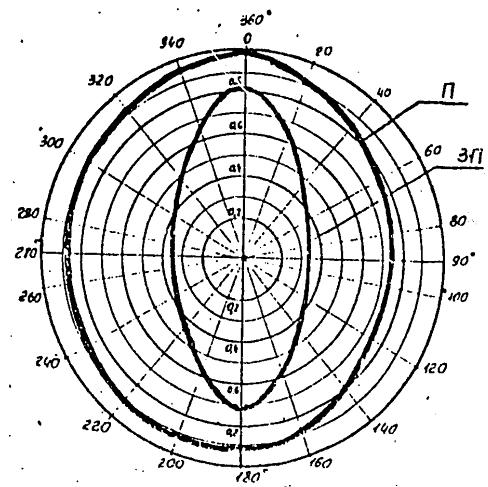

Figure 4. Probability of correct detection of a radio pulse lasting 60 nanoseconds and a filling frequency of 1 kilohertz. П -- threshold of the measuring device.

It has been established that the primary role in the formation of the binaural channel of vertebrates is played by the neuron systems of the stem structures of the brain; here the directional characteristics of the auditory system on this level differs significantly from circular even when $d/\lambda \ll 1$ (d is the base dimension);

It is demonstrated that the differences in the threshold signals, depending on the irradiation angle, is 15 to 20 decibels; the measurement of the probability of correct responses reaches 5 to 7 decibels. When measuring the fright reactions, the directional characteristics have symmetric shape.

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MARINE GEOACOUSTICS INFORMATION MEASURING SYSTEM

By O. A. Vorob'yev, V. B. Podshuveyt, pp 158-161

An effort is made below to construct a generalized, conceptual mathematical model of marine geoacoustics as a united information measuring system. The identification of complex systems implies statistical description both in view of the inhomogeneity of the elements of the system, including, in particular, man, and as a result of the presence of noise and interference in all of the subsystems.

Out of the set of feedback links, the feedback closing the output information J of block 6 in the form of the control input ∂J to the initial block 1 is the most important: this is the specific nature of geological exploration on the whole and geophysics as the leading part of it in the exploitation of the resources of the world ocean. Blocks 1, 3 and 4 are executed on observation platforms (as a rule, on board a ship); blocks 5, 7 and 6 basically, on shore. The communications between the blocks are realized either by physical transportation or transmission over telemetric channels. If all the communications in the system depicted in Figure 1 are considered adaptive, then the information process is described by a system of recurrent operator equations:

$$\begin{aligned} &1.\ (S_1+N_1+\delta_1 S_4+\delta_1 J_{K-1})*(1+K_2)+N_2=S_2,\quad 2.\ S_2*K_3+N_3=S_3,\quad 3.\ S_3*K_4+N_4=S_4,\\ &4.\ (S_4+\delta_2 J_{K-1}+\sum_{0}^{K-2} J_m)*K_3+N_3=S_5,\quad 5.\ (S_5+\delta_2 S_4)*K_6+N_6=J_K,\\ &6.\ J_0=const. \end{aligned} \tag{1}$$

Here S_i are the signals, J is the output information of the system, k_i are the operators (the transmission functions) of the blocks, N_i are the noise characteristics of the blocks, δ is the variation operator of the signals and information; the analogous feedback coefficient. The simple "*" denotes operator convolution, and "+", generalized superposition. The initial level 6 depicts the postulates adopted regarding the emitted object. For the possibility of simulation of a marine geoacoustics system on the basis of model (1) on a computer it is necessary to assign the actual meaning to blocks 1-7. In view of the complexity of the problem (each of the blocks is itself a complex system) the description of the blocks has a stipulative nature.

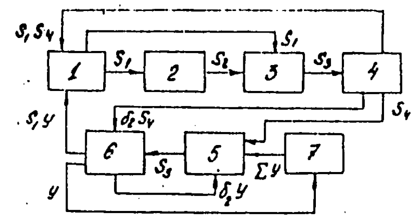

Figure 1. Block diagram of the functional structure of a sensometry marine geoacoustics information measuring system: 1 -- field source, 2 -- emitted object, 3 -- receiver, 4 -- recording equipment, 5 -- processing, storage and exchange system, 6 -- administrative-technological control, 7 -- a priori information.

Block 1. The field source: previously explosives were used: at the present time in practice the transition has been made to sources which are safe for ichthyofauna -- pneumatic emitters, sparkers; interest has increased in the use of natural seismic fields (the noise of the wavy surface [2], volcanic and tectophysical microseisms [3]). The energy of the pneumatic emitters with respect to level 0.7 is concentrated in the range of 10-80 hertz; their power reaches 10 kilowatts as with a signal duration on the order of 0.1 seconds. The noise N_1 with respect to all parameters (the position of the emitter, the moment errors, the duration, shape, directivity and radiation power) is close to gaussian.

Block 2. The operator K_2 in the investigations is the desired operator. In the direct problems of seismics usually a nonuniform horizontal layered model is used with participation in the beam approximation [4]; here the medium is assumed to be linear and stationary. The stochastic nature of the medium is insured by the fact that the dissipative-elastic parameters are given in the spectral form on the basis of the generalized Hook-Boltzmann law and also by the fact that the boundary "flux" is considered to b Erlang (most frequently, Poisson). On the basis of the adopted restrictions on the operator $(1 + K_2)$, for S_2 without feedback the following is true [4]:

$$S_2(t) = \int_0^t (S_1 + N_1)[1 + K_2(t-s)]ds. \qquad (2)$$

The first type integral equation (2) defining the Erlang flux of the elementary seismic recording is the base for solving the direct and inverse problems of marine geoacoustics.

Block 3. Receivers; autonomous (bottom, drift) and towed receivers are used; the latter are basic and are in the form of hose type oil-filled piezobraids on vibration resistant pressure gages predominately PDS-7 and PDS-21 type. The operator K_3 in the frequency range is defined by the expressions (6):

$$1.\ \dot{S}_3(\omega)=\sum_i S_{3i}(\omega);\ S_{3i}(\omega)=S_{2i}(\omega)K_{3i}(\omega/v_{ki}),$$
$$2.\ K_{3i}(\omega/v_{ki})=\sum_k D_k(\omega)\exp\{-j\omega k\Delta x/v_{ki}\} \qquad (3)$$

Here i is the group number, k is the sensor number in the group, v_{ki} is the apparent velocity of the signal on the piezobraid, D_k is the sensitivity of the kth sensor in the group. Thus, K_3 is defined as the first approximation for each group as direction characteristic K_{3i}. There are from 12 to 48 groups in the piezobraid; the sensitivity distribution of the sensors in the group, as a rule, is close to triangular with zeros K_3 at the repeated peaks [5]; there are from 50 to 100 sensors in the group. The equivalent electrical diagrams which must be taken into account in (3) are defined in [6]. In (3) the noise of the piezobraid N_3 described in detain in [6] was also not taken into account.

Block 4. The universal transition to the SSTs type digital seismic stations and universal digital geophysical information storage elements of the Grad type has been made; the elementary seismic recording on them is a digitalized (every 2 to 4 milliseconds) echo lasting from 4 to 12 seconds. The operators K_4 were investigated in detail in [4] and, as a rule, are linear.

Block 5. Processing of the recordings, the annual flow of which through NPO YuZhMORGEO alone reaches 10^5 Mbytes, is realized on the basis of a computer in the production modes. The branch set of algorithms and standard programs includes tens of statistical procedures (in addition to the usual ones, the adaptive procedures for selecting the effective recording parameters and determination of the velocity characteristic of the time section). In sum, on the basis of the Eikonal equation basically kinematic information is extracted about the time and velocity field of the elastic waves on the object. Let us note that the basic goal of processing marine geoacoustic data consists in correlation of the echoes with respect to the set of channels, profiles and areas on the object.

Block 6. Naturally block 6 is the least formalized; the determination of K_6 and N_6 is beyond the scope of the given paper.

It appears that the approach defined above can serve as the base for computer simulation of the behavior of a marine geoacoustic system as a whole and can find application when solving analogous problems in geoacoustic research.

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SOME CRITERIA OF EFFECTIVENESS OF ORGANIZATION OF MARINE SEISMOACOUSTIC RESEARCH

By O. P. Alekseyev, O. A. Vorob'yev, pp 161-162

The organization of marine seismoacoustic research (MSAR) is connected with the solution of the problems of planning and performing research in different parts of the world ocean with minimum a priori information about the investigated subject. Seismoacoustic studies in the ocean performed on board a scientific research ship (to 10^6 bits/sec), the gathering and processing of large volumes of data are giving rise to the use of complex sets of technical means located both on board scientific research ships and at shore computer centers [1]. The optimization of the organizational and technical structure of the performance of MSAR is connected with estimating the effectiveness of the research by a set of criteria taking into account both the nature of the research and the technical-economic indexes of the structure as a whole. When organizing MSAR in various parts of the world ocean, the following versions of the structures are possible: gathering data on the scientific research ship and processing at the shore computer center, gathering and transmitting data from the scientific research ship to the shore computer center over communications channels, gathering and complete processing of the data on the scientific research ship (atonomous structure). For comparative estimation of the effectiveness of the various versions of the structures it is expedient to use a model of the following type:

$$Э_j = \mathrm{ext} \frac{1}{m} \sum_{i=1}^{m} \left(\frac{1-L_i}{1+L_i} \right)^{\mu_i}, \quad j = \mathrm{opt}, \quad \begin{matrix} i = 1, \ldots, m \\ \ell = 1, \ldots, n \end{matrix}$$

where m is the number of local indexes by which an estimate is made of the jth version of the solution; n is the number of alternatives; L_i is the normalized usefulness function with respect to the ℓth criterion in the jth version; μ_i is the significance or weight of the ith quality index or local criterion.

When synthesizing a complex of technical means the usefulness function L_i can be determined by giving the quantitative form of the partial criteria

K_i, $L_i = f(K_i)$. The recording of the type of $f(K_i)$ can be made by the technical indexes of the functions of the saturated type (for cost indexes, linearly decreasing).

Since for the majority of components of the above model it is difficult to obtain a quantitative interpretation of the components, it is expedient for its realization to use expert methods, especially when determining the weights μ_i. The use of the compositional weight $\mu_i = \mu_{i1} - \mu_{i2}$ is possible where μ_{i1} is the expert evaluation of the significance of the ith criterion; μ_{i2} is the correction coefficient determined by the variation of L_i in the range of k_i: $\mu_{i2} = \partial L_i/\partial K_i$. The summation of the values of the individual indexes informative in the range of (0.1) with the corresponding weights permits a sufficiently realistic estimate to be made of the possibilities of the various versions of the marine complexes of technical means. The analysis of the actual structures of the presented model made it possible to recommend the use of an autonomous structure (gathering and processing on the scientific research ship), at significant (more than 2000 km) distance of the scientific research ship from the shore computer center. The structure with processing of the data at the shore computer centers is expedient when working in nearby waters.

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UDC 621.391.16

RESPONSE CHARACTERISTICS IN A SYSTEM OF QUASIMATCHED FILTERS WITH JOINT CONSIDERATION OF THE DOPPLER VELOCITY AND ACCELERATION

By Ye. B. Libenson, pp 162-163

In this paper a study is made of the parameters of the basic response lobe in the channels of a system of quasimatched filters with joint consideration of the doppler velocity and acceleration for complex signals with nonlinear frequency modulation (NFM).

It is demonstrated that for signals of great complexity with NFM there is a possibility for decreasing losses at the output of the matched filter in the case of distortion of the modulating function of the signal as the result of the effect of acceleration using the frequency bias of the reference signal or, at the same time, in the system of quasimatched filters. Relations were obtained which characterize the joint tolerance (and resolution) of the velocity and acceleration in the system of quasimatched filters for signals with NFM of the general type. It is demonstrated that the joint tolerance on the velocity and acceleration of an individual channel in a system of quasimatched filters coincides(in the approximation of the stationary phase method) with that for a single matched filter. This makes it possible to carry over the properties of the latter on the level of control of the degree of tolerance (or resolution) to the channel characteristics of the doppler system.

UDC 621.391.1

ESTIMATION OF THE PARAMETERS USING THE STRUCTURAL RELATIONS OF THE SIGNAL -- ABSTRACT OF THE REPORT

By O. E. Kangur, p 163

In this paper a study is made of a class of problems of estimating the signal parameters in the presence of disturbing parameters, and it is proposed that transformations of a special type be introduced which will permit achievement of relative invariance of the estimates of the useful information parameters with respect to the values of the interfering parameters.

The proposed method of estimation, just as the ordinary least squares method does not require any a priori statistics. If the a priori information is available, it can be used to improve the estimates approximately as in the ordinary least squares method. Here, however, the properties of statistical optimalness of the estimates by the least squares method are not satisfied in the general case, for the introduction of the transformation changes the noise statistics, and for nonlinear transformation the noise becomes nonadditive. The larger the signal-to-noise ratio, the less the estimate of the signal parameter $\vec{\alpha}^*$ depends on the value of $\vec{\gamma}$; at the limit this dependence disappears.

The described method of estimation was used to solve various problems -- estimation of the amplitude, frequency and phase of harmonic oscillations, estimation of the damping decrement, distinguishing FM signals against a backgroun of concentrated interference, and so on. The results of the analysis and the simulation of certain algorithms can be found in [1, 2].

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UDC 534.883.2;519.272

TIME CORRELATION FUNCTION OF THE RESPONSES OF ACOUSTIC ANTENNAS LOCATED IN A NOISE FIELD

By N. N. Vishnyakova, V. A. Geranin, V. S. Gorbenko, V. I. Koneva, L. Ya. Taradanov, pp 164-166

Let us place directional receiving antennas with the weight functions $W_p(t)$ and $W_q(t)$ at the points p and q.

We shall consider that: 1) the acoustic field $X(t, \vec{r})$ is represented by a superposition of plane waves; 2) the waves within the limits of the solid angle Ω are summed at the points p and q.

The mutual correlation function of the processes at the outputs of continuous antennas assumed to be transparent

$$K(t_1,t_2)=\iint_{PQ} K_a(t_1,t_2:\vec{r}_p,\vec{r}_q)\,d\vec{r}_p\,d\vec{r}_q, \tag{1}$$

where integration is performed over the region P for the first antenna and the region Q for the second.

$$K_a(t_1,t_2,\vec{r}_p,\vec{r}_q)=\iint_{-\infty}^{\infty} W_p(\tau_p)W_q(\tau_q)\times K_x(t_1-\tau_p,\,t_2-\tau_q:\vec{r}_p,\vec{r}_q)\,d\tau_p\,d\tau_q \tag{2}$$

or in spectral language

$$K_a(t_1,t_2:\vec{r}_p,\vec{r}_q)=(2\pi)^{-2}\iiiint_{\Omega\,-\infty}^{\infty} H_p(\omega_1)H_q^*(\omega_2)\cdot G_x(\omega_1,\omega_2:\vec{e}_1,\vec{e}_2:\vec{r}_p,\vec{r}_q)\exp[j\omega_2(t_2-\frac{\vec{r}_q\vec{e}_2}{c})-j\omega_1(t_1-\frac{\vec{r}_p\vec{e}_1}{c})]\,d\omega_1\,d\omega_2\,d\vec{e}_1\,d\vec{e}_2. \tag{3}$$

Here $K_x(t_1,t_2,\vec{r}_p,\vec{r}_q)$ is thetime-space correlation function of the free field $X(t, \vec{r})$:

$G_x(\omega_1,\omega_2:\vec{e}_1,\vec{e}_2:\vec{r}_p,\vec{r}_q)$ is the frequency-angular-spatial spectrum of the field;

$W_i(t)H_i(\omega)$ is the weight function and the complex frequency characteristic of the antenna (i = p, q).

If the field is stationary, then

$$G_x(\omega_1,\omega_2;\vec{e}_1,\vec{e}_2;\vec{z}_p,\vec{z}_q)=G(\omega_1;\vec{e}_1,\vec{e}_2,\vec{z}_p,\vec{z}_q)\delta(\omega_2-\omega_1), \tag{4}$$

$$K_a(\tau;\vec{z}_p,\vec{z}_q)=(2\pi)^{-1}\iint_{\Omega}\int_{-\infty}^{\infty}H_p(\omega)H_q^*(\omega)G(\omega;\vec{e}_1,\vec{e}_2;\vec{z}_p,\vec{z}_q)\times$$
$$\times\exp[j\omega(\tau+z_p\vec{e}_1-z_q\vec{e}_2)]d\omega d\vec{e}_1 d\vec{e}_2, \qquad (\tau=t_2-t_1) \tag{5}$$

If, in addition, there is no correlation with respect to directions, then

$$G(\omega;\vec{e}_1,\vec{e}_2;\vec{z}_p,\vec{z}_q)=G(\omega;\vec{e}_1,\vec{z}_p,\vec{z}_q)\delta(\vec{e}_2-\vec{e}_1) \tag{6}$$

and

$$K_a(\tau;\vec{z}_{pq})=(2\pi)^{-1}\int_{\Omega}H_p(\omega)H_q^*(\omega)G(\omega;\vec{z}_p,\vec{z}_q,\vec{e})\times$$
$$\times\exp[j\omega(\tau+\frac{z_{pq}\vec{e}}{c})]d\omega d\vec{e},\ (\vec{z}_{pq}=z_p-\vec{z}_q). \tag{7}$$

Assuming that the field spectrum does not depend on the coordinates of the observation points and is separable with respect to ω and $\vec{e}$;

$$G(\omega;\vec{e},\vec{z}_p,\vec{z}_q)=q(\omega)L(\vec{e}) \tag{8}$$

we obtain

$$K_a(\tau,\vec{z}_{pq})=\int_{-\infty}^{\infty}H(\omega)H_q^*(\omega)q(\omega)R(\omega;\vec{z}_{pq})e^{j\omega t}d\omega, \tag{9}$$

where

$$R(\omega,\vec{z}_{pq})=\int_{\Omega}L(\vec{e})\exp(j\frac{\omega}{c}\vec{z}_{pq}\vec{e})d\vec{e}. \tag{10}$$

If $L(\vec{e})$ = 1 (isotropic noise),

$$K_a(\tau,z_{pq})=2\int_{-\infty}H_p(\omega)H_q(\omega)q(\omega)\times$$
$$\times S_a(\frac{\omega}{c}z_{pq})\exp(j\omega\tau)d\omega,\ (z_{pq}=|\vec{z}_{pq}|). \tag{11}$$

Let us consider an antenna with equal amplitude distributions

$$H_p(\omega)=H_q^*(\omega)=A. \tag{12}$$

Here

$$K_a(\tau;z_{pq})=4A^2\int_0^{\infty}q(\omega)S_a(\frac{\omega}{c}z_{pq})\cos\omega\tau d\omega. \tag{13}$$

In the case of $q(\omega) = q_0$ (spatial white noise), the mutual correlation function of the antenna responses has the form

$$K(\tau)=\begin{cases} 2\pi A^2 g_0 c \iint\limits_{PQ} \dfrac{d\bar{z}_p\, d\bar{z}_q}{z_{pq}}, & \text{for } |\tau| < \dfrac{z_{pq}}{c}; \\ \pi A^2 g_0 c \iint\limits_{PQ} \dfrac{d\bar{z}_p\, d\bar{z}_q}{z_{pq}}, & \text{for } \tau = \dfrac{z_{pq}}{c}; \\ 0, & \text{for } |\tau| > \dfrac{z_{pq}}{c} \end{cases} \tag{14}$$

Now let us consider the special case of a continuous linear, transparent equi-amplitude antenna of length 2L divided into two located in the isotropic noise field. From expressions (1) and (13) we have:

$$K(\tau)=4A^2\int\limits_0^L\int\limits_L^{2L}\int\limits_0^\infty g(\omega)S_a\left[\frac{\omega}{c}(z_1-z_2)\right]\cos\omega\tau\, dz_1\, dz_2\, d\omega \tag{15}$$

or

$$K(\tau)=4Ac^2\int\limits_0^\infty \frac{g(\omega)}{\omega^2}\left\{1+\cos\frac{2\omega L}{c}-2\cos\frac{\omega L}{c}+\frac{2\omega L}{c}\left[S_i\left(\frac{2\omega L}{c}\right)-S_i\left(\frac{\omega L}{c}\right)\right]\right\}\cos\omega\tau\, d\omega. \tag{16}$$

Let $g(\omega) = g_0$, then $g(\omega)=g_0$,

$$K(\tau)=\begin{cases} 2\pi A^2 G_0 c\,(2L\ln 2 - c|\tau|), & \text{for } c|\tau|<L, \\ 2\pi A^2 G_0 c\left[2L\ln\dfrac{2L}{c|\tau|}-(2L-c|\tau|)\right], & \text{for } L<c|\tau|<2L, \\ 0, & \text{for } c|\tau|>2L. \end{cases} \tag{17}$$

If

$$g(\omega)=g_0\,\mathrm{rect}\left(\frac{\omega-\omega_0}{\Delta\omega}\right), \tag{18}$$

then for $\frac{\Delta\omega}{\omega_0} \ll 1$

$$K(\tau)=\frac{8A^2Lcg_0\Delta\omega}{\omega_0}\left[\left(2\frac{\omega_0L}{c}\right)^{-1}+\frac{\cos\frac{\omega_0L}{c}}{2\frac{\omega_0L}{c}}-\frac{\cos\frac{\omega_0L}{c}}{\frac{\omega_0L}{c}}+Si\left(\frac{2\omega_0L}{c}\right)-Si\left(\frac{\omega_0L}{c}\right)\right]. \tag{19}$$

The asymptotic mutual correlation functions (19) are as follows:

a/ $\omega_0 L \ll 1$

$$K(\tau)=8A^2L^2g_0\Delta\omega\cos\omega_0\tau\ ; \tag{20}$$

б/ $\frac{\omega_0 L}{c} \gg 1$

$$K(\tau)=\frac{4A^2g_0\Delta\omega}{\omega^2}\left(1+\cos\frac{2\omega_0L}{c}-\cos\frac{\omega_0L}{c}\right)\cos\omega_0\tau. \tag{21}$$

The results obtained can turn out to be useful for engineering calculations and for synthesis of hydroacoustic systems receiving hydraulic information from the antenna leads.

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